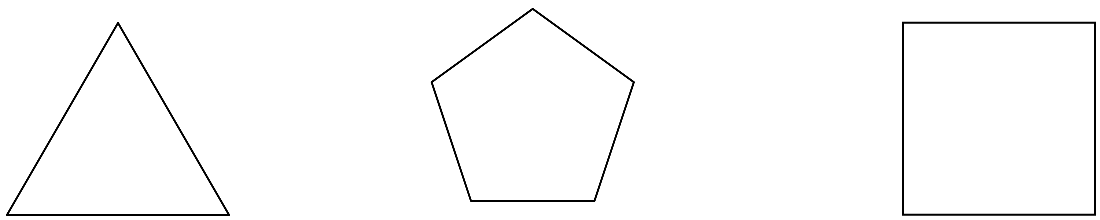
Math 1312
Section 8.2
Regular Polygons and Area

Definition:

A **regular polygon** is a polygon that is both equilateral (all sides are congruent) and equiangular (all angles are congruent).

Example1:



Theorem 1: The measure I of each interior angle of a regular polygon of n sides is

$$I = \frac{(n-2)\cdot 180^\circ}{n}.$$

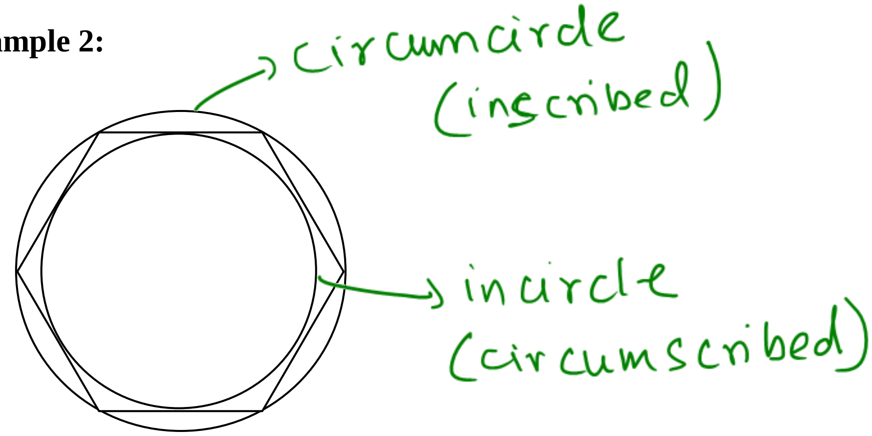
Definitions:

A polygon is **circumscribed about** a circle if all of its vertices lie on the circle.

A polygon is **inscribed in** a circle if each of its sides is tangent to the circle.

The **center of a regular polygon** is the common center for the inscribed and circumscribed circles of the polygon.

Example 2:



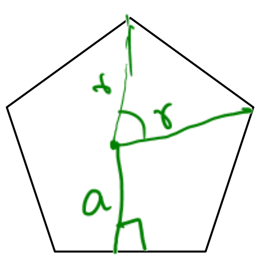
Definitions:

The segment from the center of a regular polygon perpendicular to a side of a regular polygon is called an **apothem**. (a)

The segment from the center to a vertex a regular polygon is the **radius** of the regular polygon.

A **central angle** of a regular polygon is the angle formed by two consecutive radii.

Example 3:

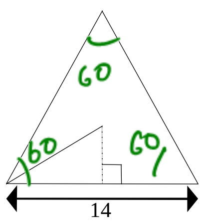


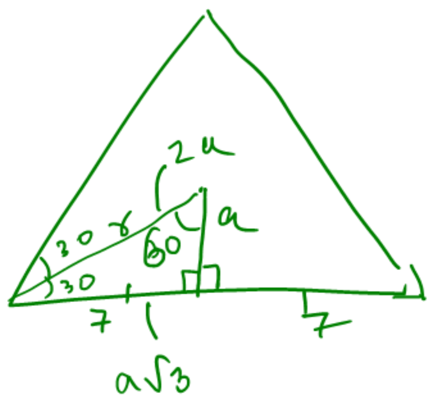
Theorem 2: The measure of each central angle is found by: $C = \dfrac{360}{n}$

Theorem 3: Any radius of a regular polygon bisects the angle at the vertex to which it is drawn and any apothem bisects the side to which it is drawn.

Theorem 4: The area of any regular polygon can be found by: $A = \dfrac{1}{2}aP$, where a = apothem and P = perimeter.

Example 4: Find the apothem (a), area (A), and perimeter (P) of each regular polygon.





$a\sqrt{3} = 7$

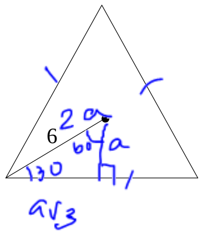
$a = \dfrac{7}{\sqrt{3}} = \dfrac{7\sqrt{3}}{\sqrt{3}\sqrt{3}} = \dfrac{7}{3}\sqrt{3}$ units

$P = 3(14) = 42$ units

$A = \dfrac{1}{2}aP = \dfrac{1}{\cancel{2}} \cdot \dfrac{7}{\cancel{3}}\sqrt{3} \cdot \cancel{42}^{\cancel{21}\ 7} = 49\sqrt{3}$ sq units

$2a = 6$

$a = 3$ units



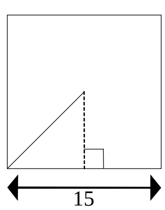
$a\sqrt{3} = 3\sqrt{3}$

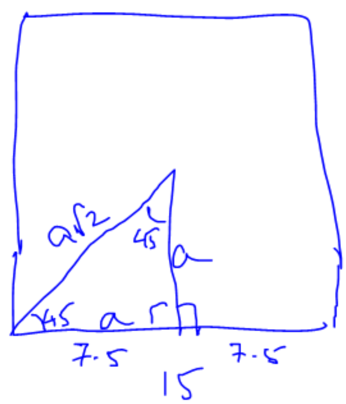
$\therefore$ side $= 2a\sqrt{3}$

$= 2(3\sqrt{3}) = 6\sqrt{3}$ units

Perimeter $= 3(6\sqrt{3}) = 18\sqrt{3}$ units

Area $= \frac{1}{2}aP = \frac{1}{\cancel{2}}\, 3\,(\overset{9}{\cancel{18}}\sqrt{3}) = 27\sqrt{3}$ sq units





$a = 7.5$ (apothem) [radius $= 7.5\sqrt{2}$]

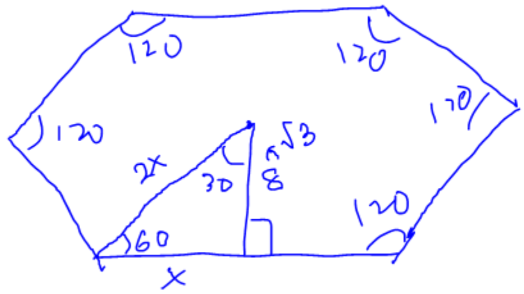
Perimeter $= 4(15) = 60$ units

Area $= \frac{1}{2}aP = \frac{1}{\cancel{2}}(7.5)(\overset{30}{\cancel{60}}) = 225$ sq units

[since this is a square $A = (15)^2 = 225$ sq units]

Example 5: Find the apothem (a), area (A), and perimeter (P) of each regular polygon.

a) Hexagon with a=8



$\left[\text{radius} = \frac{16}{3}\sqrt{3}\right]$

$$I = \frac{(n-2)180}{n} = \frac{(6-2)\overset{30}{\cancel{180}}}{\cancel{6}} = 4(30) = 120$$

$$x\sqrt{3} = 8 \Rightarrow x = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{8}{3}\sqrt{3}$$

$$\text{side} = (2x) = 2\left(\frac{8}{3}\sqrt{3}\right) = \frac{16}{3}\sqrt{3} \text{ units}$$

$$\text{Perimeter} = \overset{2}{\cancel{6}}\left(\frac{16}{\cancel{3}}\sqrt{3}\right) = 32\sqrt{3} \text{ units}$$

$$\text{Area} = \frac{1}{2}aP = \frac{1}{\cancel{2}}\overset{4}{\cancel{8}}(32\sqrt{3}) = 128\sqrt{3} \text{ sq units}$$

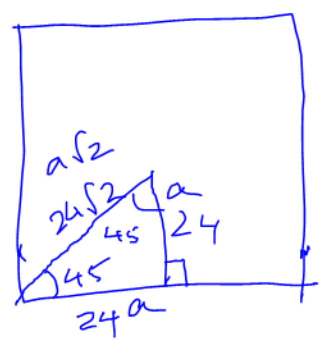
b) Octagon with apothem = 4.8, side = 4

Perimeter = 8(4) = 32 units

$$\text{Area} = \frac{1}{2}aP = \frac{1}{\cancel{2}}(4.8)\,(\overset{16}{\cancel{32}})$$

$$= 76.8 \text{ sq units}$$

c) Square with apothem = 24



apothem (a) = 24 units

[radius = $24\sqrt{2}$ units]

Side = 2a

= 2(24) = 48 units

Perimeter = 4(48) = 192 units

$$\text{Area} = \frac{1}{2}aP = \frac{1}{2}(24)(192) = 2304 \text{ sq units}$$