**Definitions:**

A **circle** (symbol O) is the set of all points in a plane that are at the same distance from the center.
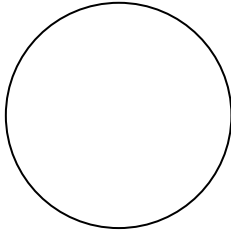


The **diameter** is a chord through the center of a circle.

The **diameter** is the distance across the circle.

The **circumference** of a circle is the distance around the circle.

**Definition:** $\pi$ is a constant equal to 3.14 or 3.1416 or $\dfrac{22}{7}$.

**Theorem 1:** The circumference of a circle is given by the formula $C = \pi d$ or $C = 2\pi r$.

**Definition:** The length of an arc is the distance between the endpoints of the arc.

**Theorem 2:** In a circle whose circumference is $C$, the length $\ell$ of an arc whose degree measure is $m$ is given by $\ell = \dfrac{m}{360} \cdot C$.

**Theorem 3:** The area $A$ of a circle whose radius has length $r$ is given by $A = \pi r^2$.

**Example 1**: Find the diameter, circumference, and the area of a circle whose radius is 8 cm.

$$r = 8 \qquad d = 2r = 2(8) = 16 \text{ cm}$$
$$C = 2\pi r = 2\pi(8) = 16\pi \text{ cm}$$
$$A = \pi r^2 = \pi(8)^2 = 64\pi \text{ cm}^2$$

**Example 2**: Find the radius, the diameter, and the area of a circle whose circumference is $22\pi$ in.

$$C = 2\pi r \qquad 2\pi r = 22\pi$$

$$d = 2(r) = 2(11) \qquad r = \frac{22\pi}{2\pi} = 11 \text{ in}$$

$$= 22 \text{ in}$$

$$A = \pi r^2 = \pi(11)^2 = 121\pi \text{ in}^2$$

**Example 3**: Find the radius and circumference of a circle whose area is $49\pi$ $m^2$.

$$A = \pi r^2 \quad \pi r^2 = 49\pi \Rightarrow r^2 = 49 \Rightarrow r = \sqrt{49} = 7m$$

$$C = 2\pi r = 2\pi(7) = 14\pi m$$

$$d = 2r = 2(7) = 14 m$$

**Example 4**: Find the length of a $48^0$ arc in a circle whose diameter is 14.

$$l = \frac{m}{360} C \qquad C = \pi d = 14\pi$$

$$= \frac{\overset{2}{48}(14\pi)}{\underset{30\ 15}{360}} = \frac{28}{15}\pi$$

**Example 5**: Find the length of a $72^0$ arc in a circle whose circumference is $45\pi$.

$$l = \frac{\overset{9}{72}(45\pi)}{\underset{8}{360}} = 9\pi$$

**Example 6**: Find the radius of a circle if a $90^0$ arc has length of $6\pi$.

$$l = \frac{m}{360} C$$

$$6\pi = \frac{\overset{90}{90}(2\pi r)}{\underset{4\ 2}{360}} \Rightarrow 6\pi = \frac{\pi r}{2}$$

$$\Rightarrow 12\pi = \pi r$$

$$\Rightarrow 12 = r$$

**Example 7**: Find the exact area of the shaded region.
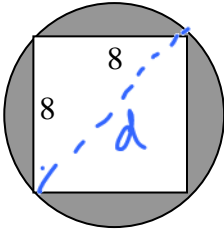
$$d^2 = 8^2 + 8^2$$
$$= 2(8^2)$$
$$d = \sqrt{2(8^2)} = 8\sqrt{2}$$

$$r = \frac{d}{2} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

$$A_S = 8^2 = 64$$
$$A_C = \pi r^2 = \pi (4\sqrt{2})^2 = \pi \, 16(2) = 32\pi$$

Area of shaded region
$$= 32\pi - 64$$

**Example 8**: Find the exact area of the shaded region (regular hexagon is inscribed in a circle of radius 6 ).