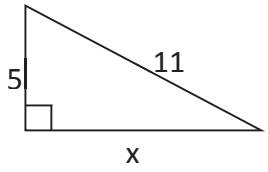
Math1312
Test 3 Review

Test 3 covers chapters 5 and 6 from the textbook.
How to study: Study the class notes, review homework problems, and try to do as many exercises as you can from the textbook. Note that answers are provided at the back of the book to all odd numbered problems.
You need to know what definitions mean and theorems and postulates as facts but you do not need to memorize them word by word.
Here I provide some examples for you. This is not a complete list, studying only these examples is not enough!

1. Find the value of x. Write your answer in the simplest radical form.

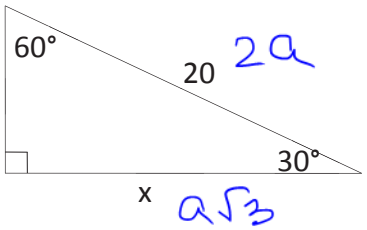
$3 \mid 96$
$2 \mid 32$
16



$5^2 + x^2 = 11^2$
$25 + x^2 = 121$
$x^2 = 96$
$x = \sqrt{96} = \sqrt{3 \cdot 2 \cdot 16} = 4\sqrt{6}$

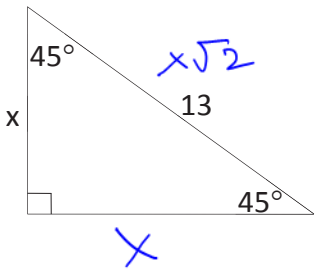
2. Find the value of x. Write your answer in the simplest radical form.

a



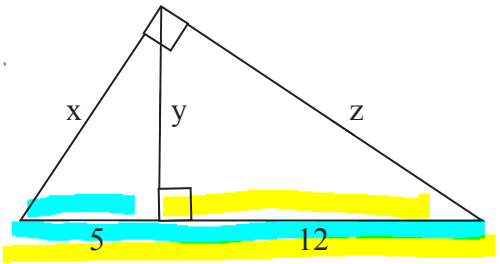
$2a = 20$
$a = 10$
$\therefore x = a\sqrt{3} = 10\sqrt{3}$

3. Find the value of x. Write your answer in the simplest radical form.



$x\sqrt{2} = 13$
$x = \frac{13}{\sqrt{2}}$
$= \frac{13\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{13\sqrt{2}}{2}$

4. Using the following figure, find x,y, and z.



y is the geometric mean of 5 & 12

$y^2 = 5 \cdot 12 \Rightarrow y^2 = 60$

$\Rightarrow y = \sqrt{60} = \sqrt{4 \cdot 15} = 2\sqrt{15}$

$5^2 + y^2 = x^2$

$25 + 60 = x^2$

$85 = x^2$

$\sqrt{85} = x$

$x^2 = 5(5+12)$

$= 5 \cdot 17$

$x^2 = 85$

$x = \sqrt{85}$

$y^2 + 12^2 = z^2$

$60 + 144 = z^2$

$204 = z^2$

$\sqrt{204} = z$

$\sqrt{4 \cdot 51} = z$

$2\sqrt{51} = z$

$z^2 = 12(12+5)$

$= 12 \cdot 17$

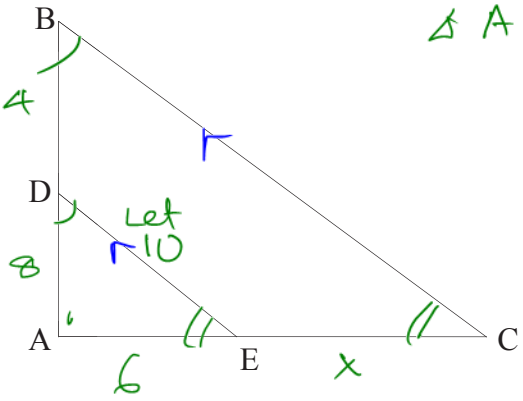
$z = \sqrt{12 \cdot 17}$

$= \sqrt{4 \cdot 3 \cdot 17}$

$= 2\sqrt{51}$

5. Given $\overline{DE} \parallel \overline{BC}$, $AD = 8$, $BD = 4$, and $AE = 6$, find CE.

BC?



$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$

$\frac{8}{12} = \frac{10}{BC}$

$BC = \frac{10 \cdot 12}{8}$

$= 15$

$\triangle ADE \sim \triangle ABC$

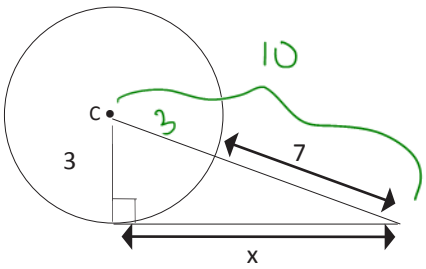
$\frac{AE}{EC} = \frac{AD}{BD}$

$\frac{6}{x} = \frac{8}{4}$

$6 \cdot 4 = 8 \cdot x$

$\frac{6 \cdot 4}{8} = x \Rightarrow 3 = x$

6. Find the value of x in the circle below.



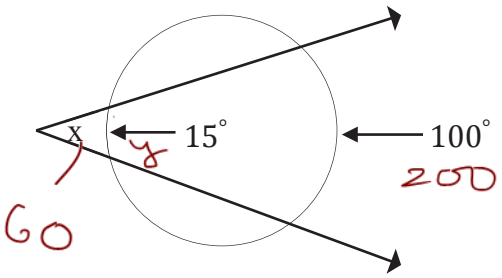
$3^2 + x^2 = 10^2$

$9 + x^2 = 100$

$x^2 = 91$

$x = \sqrt{91}$

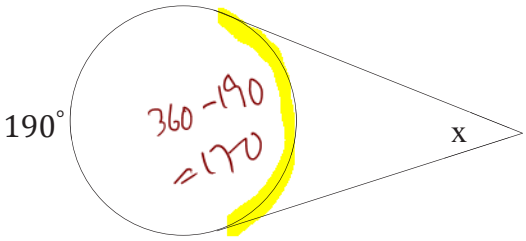
7. Find the value of x in the circle below.



$x = \frac{1}{2}(100 - 15)$

$= \frac{1}{2}(85) = 42.5^\circ$

8. Find the value of x.



$360 - 190 = 170$

$60 = \frac{1}{2}(200 - y)$

$120 = (200 - y) \Rightarrow y = 80^\circ$

$x = \frac{1}{2}(190 - 170)$

$= \frac{1}{2}(20) = 10^\circ$

9. Find AC given DE = 5, BE = 16, and AE = 10.

EC?

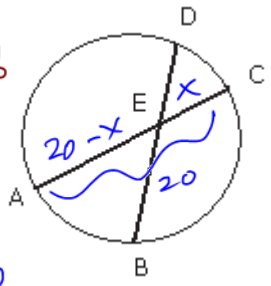
DE = 5 BE = 16

AC = 20

$(20 - x)x = 5 \cdot 16$

$20x - x^2 = 80$

$x^2 - 20x + 80 = 0$



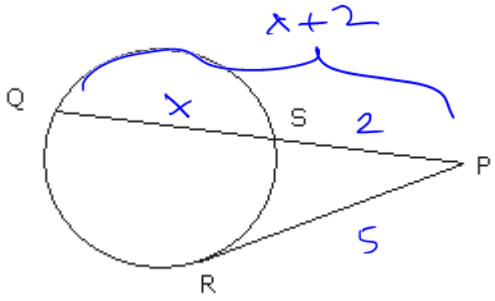
$AE \cdot EC = ED \cdot EB$

$10 \cdot EC = 5 \cdot 16$

$EC = \frac{5 \cdot 16}{10} = 8$

$\therefore AC = AE + EC = 10 + 8 = 18$

10. Given PR = 5, PS = 2, find SQ.



$RP^2 = PQ \cdot PS$

$5^2 = (x + 2)2$

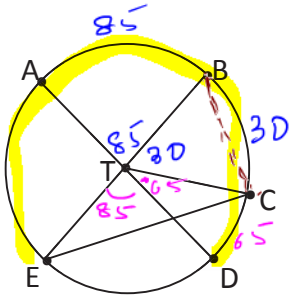
$25 = 2x + 4$

$21 = 2x$

$\frac{21}{2} = x$

$10.5 = x$

11. If $m\widehat{AB} = 85^\circ$ and $m\widehat{BC} = 30^\circ$ in a circle T, find $m\widehat{BAE}$, $m\widehat{CD}$, $m\widehat{EAD}$, $m\angle BTC$, and $m\angle BEC$.



$m\angle DTC = 180 - 85 - 30 = 65$

$m\widehat{BAE} = 180^\circ$

$m\widehat{CD} = 65^\circ$

$m\widehat{EAD} = 360 - 85 \quad (360 - m\widehat{ED})$
$= 275$

$m\widehat{EAD} = 180 + 30 + 65 \quad (m\widehat{EB} + m\widehat{BC} + m\widehat{CD})$
$= 275$

$m\angle BTC = 30$

$m\angle BEC = \frac{1}{2}(m\angle BTC)$
$= \frac{1}{2}(30) = 15^\circ$

12. Given $\Delta ABC \sim \Delta DEF$ (not shown), $m\angle A = 50^\circ$, $m\angle E = 33^\circ$, and $m\angle D = 2x + 40^\circ$. Find $x, m\angle F$.

$m\angle A = m\angle D$
$50 = 2x + 40$
$10 = 2x$
$5 = x$

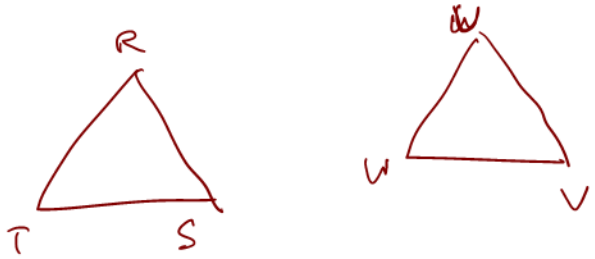
$m\angle D + m\angle E + m\angle F = 180$
$50 + 33 + m\angle F = 180$
$83 + m\angle F = 180$
$m\angle F = 180 - 83$
$= 97^\circ$

13. Given $\Delta RTS \sim \Delta UWV$ (not shown). Find WV if $RT = 4$, $UW = 6$, and $TS = 8$.

$\frac{RT}{UW} = \frac{TS}{WV} = \frac{RS}{UV} \Rightarrow \frac{4}{6} = \frac{8}{WV} \Rightarrow$

$4 \cdot WV = 8 \cdot 6$
$\therefore WV = \frac{8 \cdot 6}{4} = 12$

14. Name the method that is used to show that ΔRTS is similar to ΔUWV if $WU = 2TR, WV = 2TS$, and $UV = 2RS$.



$WU = 2TR \Rightarrow \frac{WU}{TR} = 2$

$WV = 2TS \Rightarrow \frac{WV}{TS} = 2$

$UV = 2RS \Rightarrow \frac{UV}{RS} = 2$

$\frac{WU}{TR} = \frac{WV}{TS} = \frac{UV}{RS} = 2$ SSS

$(2\sqrt{2})^2 = 2^2 \cdot (\sqrt{2})^2 = 4 \cdot 2 = 8$

15. Tell whether each set of numbers represents the lengths of the sides of an acute triangle, an obtuse triangle, of a right, or of no triangle.

a. 12, 13, 14
b. 9, 13, 8
c. $2\sqrt{2}$, $2\sqrt{3}$, 2
d. 5, 11, 7

$(2\sqrt{2})^2 = 8$ $\quad (2\sqrt{3})^2 = 12$ $\quad 2^2 = 4$

(a) $a^2 + b^2 > c^2$

$12^2 + 13^2 > 14^2$

$144 + 169 > 196$ acute

(b) $9^2 + 8^2 < 13^2$

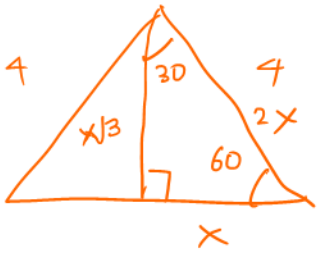
$81 + 64 = 145 < 169$ Obtuse

(c) $8 + 4 = 12$ Right

(d) $5^2 + 7^2 < 11^2$

$25 + 49 = 74 < 121$ Obtuse

16. Find the length of an altitude of an equilateral triangle if each side is 4 in long.

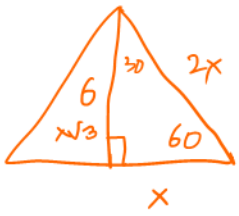


$2x = 4$

$x = 2$

Altitude $= x\sqrt{3} = 2\sqrt{3}$

17. Find the perimeter of an equilateral triangle if an altitude is 6 in long.



$x\sqrt{3} = 6$

$x = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{6\sqrt{3}}{(\sqrt{3})^2} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

Side $= 2x = 2(2\sqrt{3}) = 4\sqrt{3}$

Perimeter $= 3 \cdot 4 \cdot \sqrt{3} = 12\sqrt{3}$

18. State whether the following statement is Always True, Sometimes True, or Never True.

a) *The ratio of perimeters of two similar polygons is equal to their scale factor.*

Always

b) *The ratio of areas of two similar polygons is equal to their scale factor.*

Never true, equals to the square of the scale factor

c) *Any two rectangles are similar.*

Sometimes true, all ∠s are 90° but sides may not be proportional

d) *Any two equilateral triangles are similar.*

Always true, all ∠s 60° & sides will be proportional

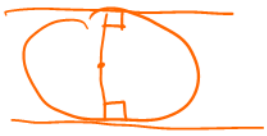
e) *A trapezoid can be inscribed in a circle.*

Sometimes true, only if the trap. is isosc.

f) *Concentric circles with different radii have no common tangents.*

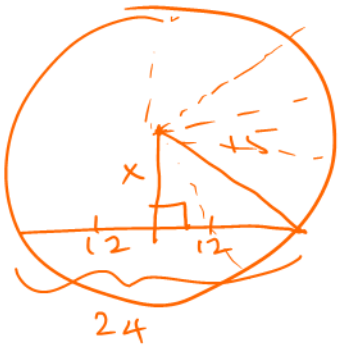
Always true

g) *Tangents to a circle at the endpoints of a diameter are parallel.*



Always true

19. The length of the radius of a circle is 15. The length of a chord is 24. Find the distance from the center of the circle to the chord.



$x^2 + 12^2 = 15^2$

$x^2 + 144 = 225$

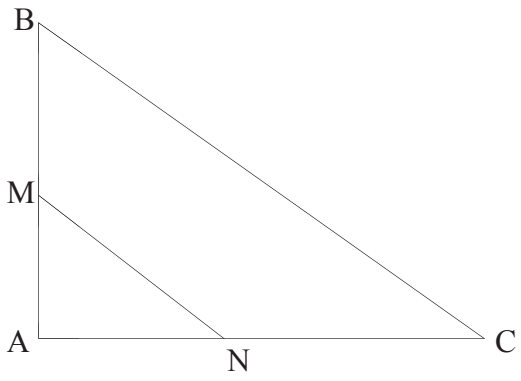
$x^2 = 81$

$x = 9$

20. Complete the following proof.

Given: ΔABC; M and N are midpoints of $\overline{AB}$ and $\overline{AC}$, respectively.

Prove: $\Delta AMN \sim \Delta ABC$



PROOF	
Statements	**Reasons**
1. ΔABC; M and N are midpoints of $\overline{AB}$ and $\overline{AC}$, respectively.	1. Given
2. $AM = \frac{1}{2}(AB)$ and $AN = \frac{1}{2}(AC)$	2. Def of mid pt
3. $MN = \frac{1}{2}(BC)$	3. Line seg that connects the mid pts of sides of a Δ is $\frac{1}{2}$ of the 3rd side
4. $\frac{AM}{AB} = \frac{1}{2}$, $\frac{AN}{AC} = \frac{1}{2}$, and $\frac{MN}{BC} = \frac{1}{2}$	4. Division
5. $\frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC}$	5. substitution
6. $\Delta AMN \sim \Delta ABC$	6. SSS

(*) Let $m\angle XYZ = 20^\circ$. Y is the mid pt of $\overset{\frown}{XYZ}$. Find $m\overset{\frown}{XY}$

$m\overset{\frown}{XYZ} = 360 - 40 = 320^\circ$

$m\overset{\frown}{XY} = \frac{1}{2} m\overset{\frown}{XYZ} = \frac{1}{2}320 = 160^\circ$