Calculus 2 Review Sheet

Question 1
Calculate the derivative of the following function:

\( f(x) = 6 \log_5 (2x) \)

a) \( f'(x) = 12 \log_5 (2x) \)

b) \( f'(x) = \frac{6}{x \ln(5)} \)

c) \( f'(x) = \frac{6}{\log_5 (2x)} \)

d) \( f'(x) = \frac{6 \ln(5)}{x} \)

e) \( f'(x) = \frac{12}{x \ln(5)} \)

Question 2
Calculate the following:

\( \frac{d}{dx} (4^{-x}) \)

a) \(-4^{-x} \ln (4)\)

b) \(4^{-x} \ln (4)\)

c) \(-x 4^{-x} \ln (4)\)

d) \(-x 4^{-x} - 1\)

e) \(\frac{4^{-x}}{\ln (4)}\)

Question 3
Calculate the following:

\( \frac{d}{dx} (\arctan(6x^2)) \)

a) \(\frac{1}{1 + 36x^4}\)

b) \(\sqrt{1 - 36x^4} \cdot \frac{12x}{12x}\)

c) \(\frac{1 + 36x^4}{2x}\)

d) \(\frac{1 + 36x^4}{6}\)

e) \(\frac{1 + 36x^4}{6}\)
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Question 4
Find the slope of the normal line to \( f(x) \) at the point where \( x = \frac{\pi}{4} \).
\[ f(x) = 2 \ln(\sec(x)) \]

a) 2  
b) -2  
c) 0  
d) \frac{1}{2}  
e) -\frac{1}{2}

Question 5
Find the slope of the normal line to \( f(x) \) at the point where \( x = 0 \).
\[ f(x) = \cosh(2x) + \sinh(3x) \]

a) 0  
b) \frac{1}{3}  
c) 3  
d) -\frac{1}{3}  
e) -3

Question 6
Find the slope of the tangent line to \( f(x) \) at the point where \( x = 0 \).
\[ f(x) = e^{2x} + \ln(3x + 1) \]

a) -\frac{1}{5}  
b) \frac{1}{5}  
c) 5  
d) \frac{1}{3}  
e) 3
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Question 7
Calculate the following:

\[ \int \frac{\cosh(4 \sqrt{x})}{\sqrt{x}} \, dx \]

a) \(-\frac{1}{8} \sinh(4 \sqrt{x}) + C\)
b) \(\frac{1}{2} \sinh(4 \sqrt{x}) + C\)
c) \(-\frac{1}{2} \sinh(4 \sqrt{x}) + C\)
d) \(\frac{1}{8} \sinh(4 \sqrt{x}) + C\)
e) \(2 \sinh(4 \sqrt{x}) + C\)

Question 8
Calculate the following:

\[ \int \frac{1}{3 \sqrt{x} (2 + \sqrt{x})} \, dx \]

a) \(-\frac{2}{3} \ln(2 + \sqrt{x}) + C\)
b) \(\frac{2}{3} \ln(2 + \sqrt{x}) + C\)
c) \(-3 \ln(3 \sqrt{x}) + C\)
d) \(-\frac{2}{3} \ln(3 \sqrt{x}) + C\)
e) \(-3 \ln(1 + \sqrt{x}) + C\)
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Question 9
Calculate the following:

\[
\int \frac{2 \ln(4x)}{x} \, dx
\]

a) \((\ln(4x))^2 + C\)
b) \(\frac{8}{\ln(4x)} + C\)
c) \(\frac{1}{4} (\ln(4x))^2 + C\)
d) \(\frac{2}{\ln(4x)} + C\)
e) \(8x \ln(4x) + C\)

Question 10

Part a: If \(f(x)\) is differentiable and invertible, \(f \, ^{(x)}\) is nonzero, and \(f(a) = b\), give a formula for \((f^{-1})\, ^{(b)}\)

Part b: Given

\(f(x) = -2x^3 - x - 5\)
verify that \(f(x)\) is invertible.

Part c: Using the function in part b, note that \(f(1) = -8\). Find \((f^{-1})\, ^{(-8)}\).

Part d: Find the equation of the tangent line to \(f^{-1}(x)\) at the point where \(x = -8\).
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Question 11

**Part a:** If \( f(x) \) is differentiable and invertible, \( f'(x) \) is nonzero, and \( f(a) = b \), give a formula for \( (f^{-1})'(b) \).

**Part b:** Given 
\[
f(x) = 3x^3 + x - 5
\]
verify that \( f(x) \) is invertible.

**Part c:** Using the function in part b, note that \( f(2) = 21 \). Find \( (f^{-1})'(21) \).

**Part d:** Find the equation of the tangent line to \( f^{-1}(x) \) at the point where \( x = 21 \).

Question 12

**Part a:** Solve \( \frac{dy}{dx} = -2y \) given that \( y(0) = 5 \).

**Part b:** A 100 liter tank initially full of water develops a leak at the bottom. Given that 10% of the water leaks out in the first 10 minutes, find the amount of water left in the tank \( t \) minutes after the leak develops if the water drains off at a rate that is proportional to the amount of water present.

Question 13

Compute the following:
\[
\int \frac{\sinh(x)}{49 + \cosh^2(x)} \, dx
\]

Question 14

Compute the following:
\[
\int \frac{x}{\sqrt{16 - x^4}} \, dx
\]

Question 15

Compute
\[
\int 3x \cos(5x) \, dx
\]
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Question 16
Compute
\[ \int 4x \ln(5x) \, dx \]

Question 17
Compute:
\[ \int \cos^4(x) \sin^3(x) \, dx \]

Question 18
Compute:
\[ \int \tan^4(x) \sec^4(x) \, dx \]

Question 19
Compute:
\[ \int \tan^3(x) \, dx \]

Question 20
Compute:
\[ \int \tan^4(x) \, dx \]