My talk is mainly based on a series of our three recent papers, where fundamental connections between wavelet frame based approach and PDE based approach (including variational and nonlinear PDE based methods) were established. In particular, connections to the total variation model, Mumford-Shah model, and anisotropic diffusions were established. The series of three papers showed that wavelet frame transforms are discretization of differential operators in both variational and PDE frameworks, and such discretization is superior to some of the traditional finite difference schemes for image restoration. This new understanding essentially merged the two seemingly unrelated areas. It also gave birth to many innovative and more effective image restoration models and algorithms.

Although the main application considered is image restoration, I will also discuss possible extensions to high-dimensional unstructured data analysis. I will present a characterization and construction of tight wavelet frames on non-flat domains in both continuum setting, i.e. on manifolds, and in discrete setting, i.e. on graphs; discuss how fast tight wavelet frame transforms can be computed and how they can be effectively used to process and analyze graph data.