Leveraging Model and Data-Driven Methods in Medical Imaging

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1 Introduction

Spectacular advances in learning-based methods in recent years have revolutionized the field of image and data analysis resulting in a major paradigm shift towards data-driven approaches. While deep learning methods were originally targeted mainly to applications from computer vision, such as object detection and object category classifications, the success of these ideas has spurred a flurry of activity in other areas of applied mathematics and engineering. Notably, recent studies have demonstrated that learning-based approaches with suitable numerical algorithms can exhibit superior performance for the solution of ill-posed inverse problems including deconvolution, denoising, deblurring, super-resolution and medical image reconstruction [4]. However, learning-based methods and especially deep learning networks typically require a very large number of training data to perform successfully – a requirement that can be highly impractical or impossible in applications such as medical imaging where access to data is often limited by physical constraints or privacy rules. Furthermore, in contrast to traditional model-driven approaches that are founded on well-understood mathematical assumptions, learning-based methods usually do not provide performance guarantees, so that one cannot predict for which problems they will outperform conventional methods and - if they do - where does the gain in performance comes from [3, 6].

Motivated by these observations, there has been an increasing interest in exploring how to combine the practical advantages of learning-based method with the theoretical understanding that comes from model-based approaches for ill-posed inverse problems. Such line of investigation includes a number of methods that take advantage of the expressive power of convolutional neural networks while imposing structural constraint in a model-driven manner to reduce the number of learnable parameters and facilitate the training process [7, 10]. Other methods apply data-driven schemes to learn the optimal parameters of classical regularized solvers and improve their performance [2]. For instance, several studies have explored ways to enhance the results of variational methods for medical image reconstruction by learning higher order statistics or by applying residual learning to remove reconstruction artifacts while preserving image structures [1, 5].

Despite these advances, major practical and theoretical questions remain [9]. Although exiting hybrid methods offer a sounder motivation than purely learned-based approaches and often yield excellent results with fewer learnable parameters, they usually do not come with provable guarantees. How to successfully integrate data and model-driven principles is still mostly heuristic. In addition, current methods for imposing constraints directly into the network architecture or for integrating learning principles into existing variation schemes and classical regularization methods have still several limitations that either hinder the ability to take full advantage of the computational power of data-driven methods or provide only partial theoretical guarantees.

2 Objectives of the workshop

This workshop aimed at gathering researchers with complementary expertise from applied mathematics, engineering and computer science to examine the impact of learning-based methods in medical image problems and discuss the most recent contributions about the integration of model- and data-driven methods for such applications. As deep learning is revolutionizing the field of image and data analysis, including medical imaging, this workshop provided an exciting and timely venue to bring together world-experts from different areas of mathematics, engineering and computer science to critically discuss state-of-the-art research in this area, present new results, identify fundamental questions and strategies to address those while engaging in open discussion and starting new collaborations.

3 Main Themes of the workshop and open problems

Within the area of methods for medical imaging, the presentations and following discussions at the workshop addressed a range of topics including both theoretical and computational aspects. Here is a list of the main topics that were covered.

1. Theoretical problems in tomographic reconstruction. This was addressed by the talks of G. Alberti and F. De Mari.
2. Tomographic reconstruction from incomplete data. Several aspects of this problems were addressed by the talks of G. Alberti, J. Frikel, B. Goossens.
3. Topics in image reconstruction from various modalities. Image reconstruction from Cryogenic Electron Microscopy was discussed by C. Esteve-Yagüe, dynamic Magnetic Resonance Imaging by J. Fessler, Electrical Impedance Tomography by A. Greeleaf and R. Murthy, Optical Tomography by M. Machida; the construction of training datasets for data-driven methods in image reconstruction was the topic of the talk of M. Kiss.
4. Deep neural networks for inverse problems. Talks addressing numerical and theoretical aspects in the applications of deep neural networks for inverse problems were delivered by T Bui-Thanh, S. Dittmer, Y. Lee, A. Mang, T. Roith and M. Santacesaria.
5. Learning-driven regularization. Talks addressing the application of learning-driven strategies in inverse problems were addressed by A. Aspri, J. Hertrich, T. Pock, C. Poon and L. Ratti

4 Presentation Highlights

Here is the list of the talks delivered during the workshop, where we use ECR = Early Career Researcher to indicates the talks by Ph.D. students.

Giovanni S. Alberti (Dept. of Mathematics, University of Genoa, Italy): *Compressed sensing for the sparse Radon transform*
Compressed sensing allows for the recovery of sparse signals from few measurements, whose number is proportional, up to logarithmic factors, to the sparsity of the unknown signal. The classical theory mostly considers either random linear measurements or subsampled isometries. In particular, the case with the subsampled Fourier transform finds applications to undersampled magnetic resonance imaging. In this talk, I will show how the theory of compressed sensing can also be rigorously applied to the sparse Radon transform, in which only a finite number of angles are considered. One of the main novelties consists in the fact that the Radon transform is associated to an ill-posed inverse problem, and the result follows from a new theory of compressed sensing for abstract inverse problems.

Andrea Aspri (Dept. of Mathematics, University of Milan, Italy): *Data driven regularization by projection*
In this talk I will speak about some recent results on the study of linear inverse problems under the premise that the forward operator is not at hand but given indirectly through some input-output training pairs. We

show that regularisation by projection and variational regularisation can be formulated by using the training data only and without making use of the forward operator. We will provide some information regarding convergence and stability of the regularized solutions. Moreover, we show, analytically and numerically, that regularisation by projection is indeed capable of learning linear operators. This is a joint work with Leon Frischauf (University of Vienna), Yury Korolev (University of Cambridge) and Otmar Scherzer (University of Vienna and RICAM).

Tan Bui-Thanh (Dept. of Aerospace Engineering and Engineering Mechanics, The University of Texas at Austin, USA): *Towards real-time solutions for inverse and imaging problems with uncertainty quantification*
Deep Learning (DL) by design is purely data-driven and in general does not require physics. This is the strength of DL but also one of its key limitations. DL methods in their original forms are not capable of respecting the underlying mathematical models or achieving desired accuracy even in big-data regimes. On the other hand, many data-driven science and engineering problems, such as inverse problems, typically have limited experimental or observational data, and DL would overfit the data in this case. Leveraging information encoded in the underlying mathematical models not only compensates missing information in low data regimes but also provides opportunities to equip DL methods with the underlying physics and hence obtaining higher accuracy. This talk introduces a Tikhonov Network (TNet) that is capable of learning Tikhonov regularized inverse problems. We rigorously show that our TNet approach can learn information encoded in the underlying mathematical models, and thus can produce consistent or equivalent inverse solutions, while naive purely data-based counterparts cannot. Furthermore, we theoretically study the error estimate between TNet and Tikhhonov inverse solutions and under which conditions they are the same. Extension to statistical inverse problems will also be presented.

Filippo De Mari (Dept. of Mathematics, University of Genoa, Italy): *Unitarization of the Radon transform*
We consider the Radon transform associated to pairs (X, Ξ), a variant of Helgason's notion of dual pair, where $X = G/K$ and $\Xi = G/H$, G being a locally compact group and K and H closed subgroups thereof. Under some technical assumptions, we prove that if the quasi regular representations of G acting on $L^2(X)$ and $L^2(\Xi)$ are irreducible, then the Radon transform admits a unitarization intertwining the two representations. If, in addition, the representations are square integrable, we provide an inversion formula for the Radon transform based on the voice transform associated to these representations. The general assumptions (in particular irreducibility and square integrability of the representations) fail in the case when X is either a noncompact symmetric space or a homogeneous tree and Ξ is the corresponding space of horocycles. Nonetheless, a unitarization theorem holds true in both cases and the outcoming unitary operator does intertwine the quasi regular representations. This is joint work with G. Alberti, F. Bartolucci, E. De Vito, M. Monti and F. Odone.

Sören Dittmer (Dept. of Mathematics, University of Bremen, Germany & DAMTP, University of Cambridge, UK): *Reinterpreting survival analysis in the universal approximator age*
In this talk, we will explore the intersection of survival analysis and deep learning. While survival analysis has been an essential part of statistics for a long time, it only recently gained attention from the deep learning community. This is likely in part due to the COVID-19 pandemic. We discuss how to fully harness the potential of survival analysis in deep learning. On the one hand, we discuss how survival analysis connects to classification and regression. On the other hand, we present technical tools: a new loss function, evaluation metrics, and the first universal approximating network that provably produces survival curves without numeric integration. We show that the loss function and model outperform other approaches on medical data and seamlessly integrate image data.

Carlos Esteve-Yagüe (Dept. of Applied Mathematics and Theoretical Physics, University of Cambridge, UK): *Spectral decomposition of atomic structures in heterogeneous cryo-EM*
In this talk I will present a recent work in collaboration with Willem Diepeveen, Ozan Öktem and Carola-Bibiane Schönlieb. We consider the problem of recovering the three-dimensional atomic structure of a flexible macromolecule from a heterogeneous cryo-EM dataset. Our method combines prior biological knowledge about the macromolecule of interest with the cryo-EM images. The goal is to determine the deformation of the atomic structure in each image with respect to a specific conformation, which is assumed to be known. The prior biological knowledge is used to parametrize the space of possible atomic structures. The parameters corresponding to each conformation are then estimated as a linear combination of the leading eigenvectors of a graph Laplacian, constructed by means of the cryo-EM dataset, which approximates the spectral properties of the manifold of conformations of the underlying macromolecule.

Jeffrey Fessler (Dept. of Electrical Engineering and Computer Science, The University of Michigan,USA): *Dynamic MRI reconstruction with locally low-rank regularizers*
Many dynamic image reconstruction problems involve models that assume that an image or image sequence satisfy low-rank or locally low-rank properties. These models often involve optimization problems involve nuclear norms or Schatten p-norms, so that the dynamics are learned from the data. Many machine learning problems like robust PCA also involve such regularizers. First-order proximal optimization methods like FISTA and POGM have worst-case convergence rates that are slower than the asymptotic convergence rates of smooth optimization algorithms like (limited memory) quasi-Newton algorithms that bring in second-order information. Furthermore, first-order methods are not easily applicable to locally low-rank models that involve regularizers that sum numerous nuclear norms of overlapping patches, because such regularizers are not prox friendly. This work-in-progress explores the use of smooth approximations to nuclear norms to facilitate gradient-based optimization methods for regularizers based on global and local low-rank models.

Jürgen Frikel (Dept. of Mathematics, University of Regensburg, Germany): *Modelling data incompleteness in tomography*
In this talk, we consider the reconstruction problem of X-ray tomography where only incomplete data are available. We review the known mathematical characterizations of limited data reconstructions and explain the impact of data incompleteness on reconstruction quality. In particular, we discuss why certain features of the searched object cannot be reconstructed reliably from incomplete tomographic data without imposing strong a priori assumptions or without the use of machine learning techniques. We will also explain why and what kind of artifacts can be generated during the reconstruction process, and how the classical models can be modified to mitigate artifact generation. Finally, we discuss how these theoretical insights can be used in practice, e.g. to design dedicated reconstruction techniques or to provide information about reliably reconstructed features.

Bart Goossens (Dept. of Telecommunications and Information Processing, Ghent University, Belgium): *Efficient region-of-interest CT reconstruction using near-orthogonal shearlet-based discrete projection transforms with effective pre- and post-conditioning schemes*
In a previous work, we introduced a CT reconstruction algorithm that leverages the robust width property to achieve high numerical accuracy under relaxed data consistency conditions. This algorithm jointly operates on projection and image data and has shown promising results. To further enhance its computational efficiency and reconstruction quality, here we investigate specific joint projection and image data transforms that are orthogonal, namely, we consider a class of composed Radon-shearlet transforms endowed with an intertwining property and having (near) orthogonal basis functions. However, when the continuous Radon transform is replaced by discrete parallel beam/fan beam projectors, orthogonality is lost. To manage this situation, we introduce an effective CG pre- and postconditioning scheme to take advantage of near-orthogonal composed transforms.

Allan Greenleaf (Dept. of Mathematics, University of Rochester, USA): *A Cubic Correction in EIT Imaging*
Virtual Hybrid Edge Detection (VHED) is a proposed method for applying analysis of complex principal type operators to voltage-to-current data in EIT. This allows one to obtain 2D images which, while still low resolution, highlight discontinuities of the electrical conductivity, and is potentially useful, e.g., in continuous monitoring of stroke patients where higher quality CT or MRI imaging is not feasible. It also appears useful for pre-processing EIT data before applying machine learning algorithms (work of Agnelli, et al.) The original VHED was based on just the linear term in a Neumann expansion of Astala-Päivärinta-type solutions; I will describe a possible improvement using third order correction terms.

Johannes Hertrich (Dept. of Mathematics, TU Berlin, Germany): (ECR talk) *The Power of Patches for Training Normalizing Flows*
In this talk we introduce two kinds of data-driven patch priors learned from very few images: First, the Wasserstein patch prior penalizes the Wasserstein-2 distance between the patch distribution of the reconstruction and a possibly small reference image. Such a reference image is available for instance when working with materials' microstructures or textures. The second regularizer learns the patch distribution using a normalizing flow. Since already a small image contains a large number of patches, this enables us to train the regularizer based on very few training images. For both regularizers, we show that they induce indeed a probability distribution such that they can be used within a Bayesian setting. We demonstrate the performance of patch priors for MAP estimation and posterior sampling within Bayesian inverse problems. For

both approaches, we observe numerically that only very few clean reference images are required to achieve high-quality results and to obtain stability with respect to small pertubations of the problem.

Maximilian Kiss (Dept. of Computational Imaging, Centrum Wiskunde & Informatica, The Netherlands): (ECR talk) *Using the 2DeteCT data collection as training data for data-driven methods in medical imaging*
Recent research in computational imaging largely focuses on developing machine learning (ML) techniques for image reconstruction which requires large-scale training datasets consisting of measurement data and ground-truth images. For many important imaging modalities such as X-ray Computed Tomography (CT), especially in the field of medical CT, suitable experimental datasets are scarce and many methods are developed and evaluated on simulated data, only. To overcome this challenge some data-driven methods employ prior knowledge through mathematical and/or physical models, others train on more abundant image training datasets such as ImageNet and use transfer learning on a smaller subset of medical training images to reach high performance. We propose the use of a more adequate large training dataset that contains 2D-CT instead of natural images. We acquired a versatile, open 2D CT dataset suitable for developing ML techniques for image reconstruction tasks such as low-dose reconstruction, limited or sparse angular sampling, beam-hardening artifact reduction, super-resolution, region-of-interest tomography or segmentation. For this we designed a sophisticated, semi-automatic scan procedure that utilizes a highly-flexible laboratory X-ray CT set-up. A diverse mix of samples with high natural variability in shape and density resembling abdominal CT scans, was scanned slice-by-slice in a 2D fan-beam geometry. Each of the 5000 slices was scanned with very high angular and spatial resolution and three different beam characteristics: A high-fidelity, a low-dose and a beam-hardening-inflicted mode. In addition, 850 out-of-distribution slices were scanned with sample and beam variations. The total scanning time was 850 hours. We provide the complete image reconstruction pipeline: raw projection data, pre-processing and reconstruction scripts using open software, and reference reconstructions and segmentations.

Yolanne Lee (Dept. of Computer Science, University College London, UK): (ECR talk) *Generalizing PINNs to Complex Geometries*
Partial differential equations (PDEs) are ubiquitous in the world around us, modelling phenomena from heat and sound to quantum systems residing in Euclidean and on complex geometries. Defining such laws implicitly through neural network architectures motivates the concept of physics-informed neural networks (PINNs), which use PDEs as soft constraints. Whilst PINNs have been proposed to solve PDEs in the non-Euclidean domain, current methods fundamentally discretize the domain. To date, there is no clear method to inform PINNs about the continuous topology of the PDE domain. Implicit neural representations (INRs) have emerged as a method to learn a continuous function on its entire domain. In this work, the INR framework is extended to propose Manifold-PINNs, a modified INR which can incorporate PINN constraints to approximate the solution of PDE on embedded manifolds within the domain.

Manabu Machida (Dept. of Informatics, Kindai University, Japan): *Nonlinear Rytov approximation as a practial inversion scheme for optical tomography*
The Rytov approximation has been commonly used for optical tomography. It is known that the Rytov approximation often gives better reconstructed images than the Born approximation. In the conventional Rytov approximation, however, nonlinear inverse problems must be linearized. In this talk, nonlinear reconstruction with the inverse Rytov series will be discussed.

Andreas Mang (Dept. of Mathematics, University of Houston, USA): *Shape classification through the lens of geodesic flows of diffeomorphisms*
We present work on statistical analysis on infinite-dimensional shape spaces $\mathcal{S}$. Our goal is to provide a mathematical framework for automatic classification and clustering of k-dimensional shapes $s \in \mathcal{S}$ in $\mathbb{R}^3$. The applications of our work are in biomedical imaging; we target the discrimination of clinically distinct patient groups through the lens of geodesic flows of diffeomorphisms. In a Riemannian setting, we can express the similarity between two shapes $s_0, s_1 \in \mathcal{S}$ in terms of an energy minimizing diffeomorphism $y \in \mathcal{Y}$ that maps s_0 to s_1, i.e., $y \cdot s_0 = s_1$. We will discuss different variational formulations to compute y, and showcase effective numerical algorithms to their solution. We will see that this is an ill-posed inverse problem, resulting in high-dimensional ill-conditioned optimality systems that are challenging to solve in an efficient way. We will assess their performance in terms of computational complexity, rate of convergence, time-to-solution, and inversion accuracy. In addition, we will assess the discriminiative power of machine

learning techniques implemented on several features derived from the computed map y to classify clinical data.

Rashmi Murthy (Dept. of Mathematics, Bangalore University, India): *Combing deep learning with the Electrical Impedance Tomography to classify stroke*
Electrical impedance tomography (EIT) is an imaging method based on probing an unknown conductive body with electrical currents. Voltages resulting from the current feeds are measured at the surface, and the conductivity distribution inside is reconstructed. This is a promising technique in medical imaging as various organs and tissues have different conductivities. The motivation of this talk arises from classifying the two different kinds of strokes in the brain, ischemic or haemorrhagic. Typical EIT images are not optimal for stroke-EIT because of blurred images. In this talk we present a neural network approach to classify the stroke using the EIT boundary measurements. Here, we first approximate the idealised boundary condition, that is Dirichlet-to-Neumann (DN) map and use this approximation of idealised DN map to extract robust features called Virtual Hybrid Edge Detection (VHED) functions that have a geometric interpretation and whose computation from EIT data does not involve calculating a full image of the conductivity. We report the measures of accuracy for the stroke prediction using VHED functions on datasets that differ from the training data used for the training of neural network.

Thomas Pock (Institute of Computer Graphics and Vision, TU Graz, Austria): *Posterior-variance-based error quantification for inverse problems in imaging*
In this work, a method for obtaining pixel-wise error bounds in Bayesian regularization of inverse imaging problems is introduced. The proposed method employs estimates of the posterior variance together with techniques from conformal prediction in order to obtain coverage guarantees for the error bounds, without making any assumption on the underlying data distribution. It is generally applicable to Bayesian regularization approaches, independent, e.g., of the concrete choice of the prior. Furthermore, the coverage guarantees can also be obtained in case only approximate sampling from the posterior is possible. With this in particular, the proposed framework is able to incorporate any learned prior in a black-box manner. Guaranteed coverage without assumptions on the underlying distributions is only achievable since the magnitude of the error bounds is, in general, unknown in advance. Nevertheless, experiments with multiple regularization approaches presented in the paper confirm that in practice, the obtained error bounds are rather tight. For realizing the numerical experiments, also a novel primal-dual Langevin algorithm for sampling from non-smooth distributions is introduced in this work.

Clarice Poon (Dept. of Mathematical Sciences, University of Bath, UK): *Smooth over-parametrized solvers for non-smooth structured optimisation*
Non-smooth optimization is a core ingredient of many imaging or machine learning pipelines. Non-smoothness encodes structural constraints on the solutions, such as sparsity, group sparsity, low-rank and sharp edges. It is also the basis for the definition of robust loss functions such as the square-root lasso. Standard approaches to deal with non-smoothness leverage either proximal splitting or coordinate descent. The effectiveness of their usage typically depend on proper parameter tuning, preconditioning or some sort of support pruning.

In this work, we advocate and study a different route. By over-parameterization and marginalising on certain variables (Variable Projection), we show how many popular non-smooth structured problems can be written as smooth optimization problems. The result is that one can then take advantage of quasi-Newton solvers such as L-BFGS and this, in practice, can lead to substantial performance gains. Another interesting aspect of our proposed solver is its efficiency when handling imaging problems that arise from fine discretizations (unlike proximal methods such as ISTA whose convergence is known to have exponential dependency on dimension). On a theoretical level, one can connect gradient descent on our over-parameterized formulation with mirror descent with a varying Hessian metric. This observation can then be used to derive dimension free convergence bounds and explains the efficiency of our method in the fine-grids regime.

Luca Ratti (Dept. of Mathematics, University of Bologna, Italy): *Learned variational regularization for linear inverse problems*
Variational regularization is a well-established technique to tackle instability of inverse problems, and it requires solving a minimization problem in which a mismatch functional is endowed with a suitable regularization term. The choice of such a functional is a crucial task, and it usually relies on theoretical suggestions as well as a priori information on the desired solution. A promising approach to this task is provided by

data-driven strategies, based on the statistical learning paradigm: supposing that the exact solution and the measurements are distributed according to a joint probability distribution, which is partially known thanks to a suitable training sample, we can take advantage of this statistical model to design operators. In this talk, I will consider linear inverse problems (associated with relevant applications, e.g., in signal processing and in medical imaging), and aim at learning the optimal regularization operator, among the ones belonging to some classes described by suitable parameters. I will first focus on the family of generalized Tikhonov regularizers, for which it is possible to prove theoretical properties of the optimal operator and error bounds for its approximation as the size of the sample grows, both with a supervised-learning strategy and with an unsupervised-learning one. Finally, I will discuss the extension to different families of regularization functionals, with a particular interest in sparsity-promotion. This is based on joint work with G. S. Alberti, E. De Vito, M. Santacesaria (University of Genoa), and M. Lassas (University of Helsinki).

Tim Roith (Dept. of Mathematics, Friedrich-Alexander University at Erlangen-Nürnberg, Germany): (ECR talk) *Bregman Iterations for sparse neural networks and architecture search*
I will present a novel learning framework based on stochastic Bregman iterations. It allows to train sparse neural networks with an inverse scale space approach, starting from a very sparse network and gradually adding significant parameters. Furthermore, I will provide a sparse parameter initialization strategy and a stochastic convergence analysis of the loss decay, and additional convergence proofs in the convex regime. It turns out, that the Bregman learning framework can also be applied to Neural Architecture Search. It can for instance, unveil an autoencoder structure for denoising or deblurring problems. This can be further applied to biomedical imaging. By additionally introducing learnable skip connections this allows to learn a U-Net like architecture.

Matteo Santacesaria (Dept. of Mathematics, University of Genoa, Italy): *Continuous generative neural networks for inverse problems*
Generative models are a large class of deep learning architectures, trained to describe a subset of a high dimensional space with a small number of parameters. Popular models include variational autoencoders, generative adversarial networks, normalizing flows and, more recently, score-based diffusion models. In the context of inverse problems, generative models can be used to model prior information on the unknown with a higher level of accuracy than classical regularization methods. In this talk we will present a new data-driven approach to solve inverse problems based on generative models. Taking inspiration from well-known convolutional architectures, we construct and explicitly characterize a class of injective generative models defined on infinite dimensional functions spaces. The construction is based on wavelet multi resolution analysis: one of the key theoretical novelties is the generalization of the strided convolution between discrete signals to an infinite dimensional setting. After an off-line training of the generative model, the proposed reconstruction method consists in an iterative scheme in the low-dimensional latent space. The main advantages are the faster iterations and the reduced ill-posedness, which is shown with new Lipschitz stability estimates. We also present numerical simulations validating the theoretical findings.

5 Outcome of the Meeting

In accord with the goals we had initially set for the workshop, the event produced the following outcomes.

- *To foster collaborations among researchers with different background and expertise.* To address the theoretical and computational challenges of this field there is a critical need of integrating theoretical and computational expertise. This workshop brought together researchers with a diverse background from optimization, inverse problem, numerical and harmonic analysis, machine/deep learning and their applications to problems from medical imaging. The workshop provided a very successful venue to discuss and advance theoretical and computational aspects concerning the integration model- and data-driven methods for the solution of such problems. The format of workshop, which included presentations, open discussions and opportunities for small-group meetings, was very effective to create a fertile environment for a fruitful discussion of theoretical, numerical and applied aspects of this investigation involving researchers from different backgrounds. To facilitate a richer and mutually beneficial interaction, our list of participants included researchers at different career levels so that junior investigators had the opportunity to interact and learn from more senior and well established researchers while at the same time bringing innovative ideas and new perspectives. Another goal of the workshop was to foster a closer interaction between different institutions and research centers, share data and start inter-institutional scientific collaborations. During this workshop, many researchers exchanges contacts, received formal or informal invitations to deliver talks at other institutions and initiated discussions for potential collaborations.

- *To generate new ideas to address fundamental questions.* While the integration of data- and model-driven methods in medical imaging offers the tantalizing opportunity to combine the best of both worlds in terms of computational performance and theoretical justification, research into achieving this objective is still in its infancy. For instance, current methods for imposing structural constraints based on model-driven principles in the network architecture are limited to low-dimensional linear inequalities or polyhedra with a small number of vertices. How to incorporate projections onto more complex constraint is still a major challenge. Additionally, while imposing model-driven constraints in a neural network can be well motivated and can improve performance, it is virtually an open question how to derive provable performance guarantees. Consistently with one main goals of the workshop, the focus of many talks was about current advances in applied mathematics and computation, which could provide the theoretical and computational premises to tackle such questions and challenges. For instance, a number of talks and discussions addressed the theoretical foundation of image reconstruction and the interpretability of data driven methods in inverse problems, leading to lively discussions. By bringing together experts with a diverse and complementary background, this meeting offered a very productive venue for proposing new ideas and strategies to address fundamental problems and advance current limitations in this area of investigation.

- *To identify future directions in medical imaging.* The impact of deep learning-based methods for medical image modalities such as Computed Tomography, Electrical Impedance Tomography, Magnetic Resonance Imaging and Cryogenic Electron Microscopy is expected to be very high and will lead to a new generation of technologies for medical imaging. This workshop has brought together several world-experts working on data- and model-based methods for medical imaging and provided a forum to discuss new strategic directions of investigations and to pose critical open questions in this area. As mentioned above in the highlights of the presentations, several talks addressed these critical problems with special focus on tomographic image reconstruction from incomplete data and how to address the chronic lack of measured data coming from medical imaging modalities; for instance, one talk featured the careful construction of a dataset resembling abdominal CT scan made of a diverse mix of samples (dried fruit and nuts) with high natural variability in shape and density [8].

In conclusion, the workshop was very successful in bringing together a well assorted combination of researchers from different backgrounds and career levels. It achieved its stated goals of fostering new collaborations, discussing state-of-the-art advances in this field and stimulating new ideas and research directions.

References

[1] Jonas Adler and Ozan Öktem. Solving ill-posed inverse problems using iterative deep neural networks. *Inverse Problems*, 33(12):124007, 2017.

[2] Giovanni S Alberti, Ernesto De Vito, Matti Lassas, Luca Ratti, and Matteo Santacesaria. Learning the optimal tikhonov regularizer for inverse problems. *Advances in Neural Information Processing Systems*, 34:25205–25216, 2021.

[3] Vegard Antun, Francesco Renna, Clarice Poon, Ben Adcock, and Anders C Hansen. On instabilities of deep learning in image reconstruction and the potential costs of AI. *Proceedings of the National Academy of Sciences*, 117(48):30088–30095, 2020.

[4] Simon Arridge, Peter Maass, Ozan Öktem, and Carola-Bibiane Schönlieb. Solving inverse problems using data-driven models. *Acta Numerica*, 28:1–174, 2019.

[5] Tatiana A Bubba, Mathilde Galinier, Matti Lassas, Marco Prato, Luca Ratti, and Samuli Siltanen. Deep neural networks for inverse problems with pseudodifferential operators: An application to limited-angle tomography. *SIAM Journal on Imaging Sciences*, 14(2):470–505, 2021.

[6] Martin Genzel, Jan Macdonald, and Maximilian März. Solving inverse problems with deep neural networks–robustness included? *IEEE transactions on pattern analysis and machine intelligence*, 45(1):1119–1134, 2022.

[7] Kerstin Hammernik, Teresa Klatzer, Erich Kobler, Michael P Recht, Daniel K Sodickson, Thomas Pock, and Florian Knoll. Learning a variational network for reconstruction of accelerated mri data. *Magnetic resonance in medicine*, 79(6):3055–3071, 2018.

[8] Maximilian B Kiss, Sophia B Coban, K Joost Batenburg, Tristan van Leeuwen, and Felix Lucka. 2detect–a large 2d expandable, trainable, experimental computed tomography dataset for machine learning. *arXiv preprint arXiv:2306.05907*, 2023.

[9] Gregory Ongie, Ajil Jalal, Christopher A Metzler, Richard G Baraniuk, Alexandros G Dimakis, and Rebecca Willett. Deep learning techniques for inverse problems in imaging. *IEEE Journal on Selected Areas in Information Theory*, 1(1):39–56, 2020.

[10] Jenny Schmalfuss, Erik Scheurer, Heng Zhao, Nikolaos Karantzas, Andrés Bruhn, and Demetrio Labate. Blind image inpainting with sparse directional filter dictionaries for lightweight CNNs. *Journal of Mathematical Imaging and Vision*, pages 1–17, 2022.