Sparse Multi-Regularized Shearlet-Network using Convex Relaxation for Face Recognition

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***Abstract*—This paper presents a novel approach for face recognition (FR) based on a new multiscale directional approach, called Shearlet Network (SN), and on a recently emerged machine learning paradigm, called Multi-Task Sparse Learning (MTSL). SN aims to extract anisotropic features from an image in order to efficiently capture the facial geometry (shearface); MTSL is used to exploit the relationships among multiple shared tasks generated by changing the regularization parameter to make the optimization convex. We compare our algorithm, called Sparse Multi-Regularized Shearlet Network (SMRSN), against different state-of-the-art methods on different experimental protocols with AR, ORL, LFW, FERET, FRGC v1 and Lab2 databases. Our tests show that the SMRSN approach yields a very competitive performance and outperforms several standard methods of FR.**

Keywords*—*Shearlet, Sparsity, Multi-Regularized Shearlet Network, Face Recognition.

I. INTRODUCTION

Face recognition (FR) is among the most challenging problems in pattern recognition and is a task of major relevance in applications of computer vision and machine learning. Many sparsity-based methods have been recently proposed such as the successful Sparse Representation-based Classification (SRC) introduced by Wright et al. [1]. In the SRC approach, the testing face image is represented as a sparse weighted combination of the training samples and the classification is based on assessing which class yields the minimal representation error. Recently, Yang et al. [2] [3] have proposed another powerful method, the Regularized Robust Coding (RRC) approach, which could robustly regress a given signal (image) with regularized regression coefficients. By assuming that the coding residual and the coding coefficient are respectively independent and identically distributed, the RRC seeks for a maximum a posterior solution of the coding problem. An iteratively reweighted regularized robust coding algorithm was proposed to solve the RRC model efficiently.

Among the methods remerged in the machine learning literature, the Multi-Task Learning (MTL) originally proposed by Caruana [4] has been especially influential. MTL attempts to learn classifiers for multiple tasks jointly and works under the assumption that all tasks should share some common features. Many variants of MTL were proposed, including the multi-stage multi-task feature learning (MSMTFL) introduced by Gong et al. [5], who defined a non-convex formulation for multi-task sparse feature learning based on a novel non-convex regularizion, called capped-ℓ_1, ℓ_1 regularized model for multi-task feature learning. This approach aims to simultaneously learn the features specific to each task as well as the common features shared among tasks. Related to this, the approach proposed by Zhang [6] [7] uses a multi-stage convex relaxation scheme for solving problems with non-convex objective functions. For learning formulations with sparse regularization, an analysis of the behavior of a specific multistage relaxation scheme was obtained.

In this paper, we propose a method called Sparse Multi-Regularized Shearlet Network (SMRSN), which combines sparsity, regularization theory and MTL. Sparsity, in particular, is based on the use of the shearlet representation, a powerful multiscale framework that is especially effective to capture directional and anisotropic features with high efficiency [8]. Our method includes a multi-regularization step inspired from multi-stage convex relaxation [6] to upgrade from a non-convex optimization to a convex relaxation. As part of this work, we have assessed the performance of the SMRSN approach for FR and successfully compared it against state-of-the-art algorithms.

The rest of this paper is organized as follows. In Sec. 2, we briefly describe the necessary background on shearlets. Sec. 3 presents the proposed Sparse Multi-Regularized Shearlet Network algorithm. In Sec. 4, we present several numerical experiments to demonstrate the efficacy of the proposed algorithm and compare it against competing algorithms. Finally, Sec. 5 concludes this paper.

II. SHEARLET

The shearlet transform, introduced by one of the authors and his collaborators in [9], is an approach where the analyzing filters are designed to capture information across several scales and efficiently encode anisotropic features such edges and other elongated discontinuities. To achieve optimal sparsity, shearlets are scaled according to a shear matrix B_s, $s \in \mathbb{R}$, and an anisotropic dilation matrix A_a, $a > 0$, defined by:

$$B_s = \begin{pmatrix} 1 & -s \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad A_a = \begin{pmatrix} a & 0 \\ 0 & \sqrt{a} \end{pmatrix}$$

Thanks to their properties, shearlets have been successfully employed in a number of image processing application including denoising, edge detection and feature extraction [10][11][12].

Unlike the classical wavelet transform which only depends on scales and translations, the shearlet transform is a function of three variables: the *scale a*, the *shear s* and the *translation t*. One of the most remarkable properties of the Continuous Shearlet Transform is its ability to detect very precisely the geometry of the singularities of a 2-dimensional function f by using highly directional filters as those shown in Figure1.

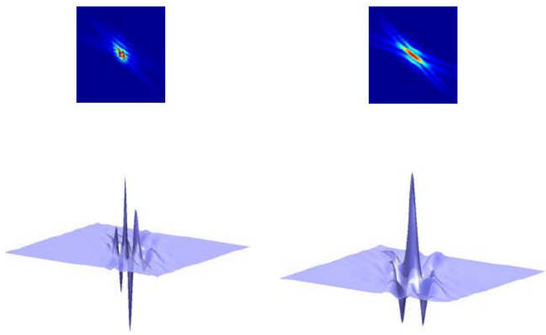

Fig. 1. Directional filters of shearlet.

By sampling the Continuous Shearlet Transform on an appropriate discrete grid, one obtains the corresponding Discrete Shearlet Transform. In this case, the discrete shearlets are functions of the form:

$$\psi_{j,l,k}(x) = 2^{\frac{3j}{2}}\psi(B_l A^j x - k) \quad j \ge 0, -2^j \le l \le 2^j - 1, k \in \mathbb{Z}^2 \qquad (1)$$

Here:

$B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $A = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$ are the shear matrix and the anisotropic dilation matrix, respectively. Note that, by choosing the generator function appropriately, the discrete shearlets form a tight frame of well-localized waveforms defined at various scales, orientations and locations.

III. SPARSE MULTI-REEGULARIZED SHEARLET NETWORK (SMRSN)

Our proposed SMRSN scheme for FR is defined as a cascade of a feature extraction module followed by a recognition module. We will implement this scheme by the use of multi-stage regularization, where the extraction of directional features is controlled by the Shearlet Network (SN), as shown in Figure 2.

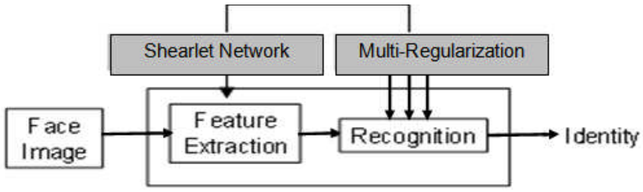

Fig. 2. SMRSN face recognition schema.

A. *Multi-Stage Regularization based Convex Relaxation*

We can model the FR problem using a standard statistical method such as Subset Selection (L_0 regularization) [7], which consists in computing the following estimator:

$$\hat{w}_{L_0} = \arg\min_{w \in R^d} \|Xw - y\|_2^2 \text{ subject to } \|w\|_0 \le \delta \qquad (2)$$

where δ is a tuning parameter, y is a normalized test face and X is a matrix representing a gallery of faces.

FR can be framed as a regression problem aimed at approximating a multivariate function from sparse data. This is an ill-posed problem and a classical way to solve it is though regularization theory [13] [14]. In practice, rather than looking for an exact solution, it is sufficient to compute an approximate one. The most popular approximation method is the regularization method which is often referred to as Lasso [15] and is given by:

$$\hat{w}_{L_1} = \arg\min_{w \in R^d} \left[\frac{1}{n} \|Xw - y\|_2^2 + \lambda \|w\|_1 \right] \qquad (3)$$

where $\lambda > 0$ is an appropriately chosen regularization parameter. However, as described in [6], this formulation is non-convex for classification problems (FR in particular). One major difficulty with non-convex formulations is that the global optimal solution cannot be efficiently computed, and the behavior of a local solution is difficult to analyze.

Convex relaxation has been commonly adopted to remedy this problem. The choice of convex formulation makes the solution unique and efficient to compute. The multi-stage convex relaxation [6] is defined as:

$$\hat{w}^{(\ell)} = \arg\min_{w \in R^d} \left[\frac{1}{n} \|Xw - y\|_2^2 + \sum_{j=1}^{d} \lambda_j^{(\ell-1)} |w_j| \right] \qquad (4)$$

where X is an $n \times d$ matrix, y an $n \times 1$ matrix, $\lambda_j^{(0)} = \lambda$, $\ell = 1, 2, \ldots$ and $j = 1, \ldots, d$. In this paper, we will adopt this formulation to assess the performance of the SMRSN approach for FR.

B. *SN for Features Extraction*

Our proposed SMRSN approach is initialized by training a shearlet network (SN) [16] [35] to models the faces. The

Gallery faces are approximated by a shearlet network to produce a compact biometric signature. A test (Probe) face is projected on the shearlet network of the Gallery face and new weights specific to this face are produced. The family of shearlets remains unchanged (this is the Gallery face).

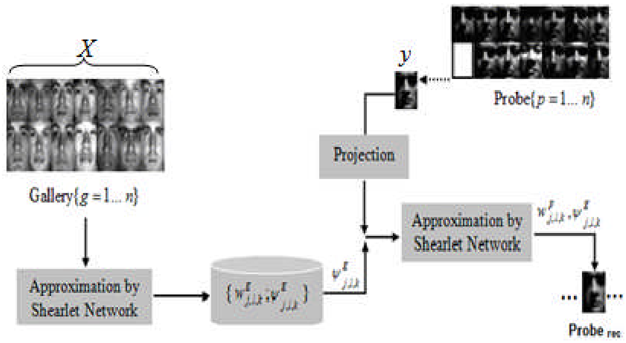

Fig. 3. Overview of SN for features extraction.

Recall that the collection of shearlets forms a tight frame, meaning that, for any image in the space of square integrable functions we have the reproducing formula:

$$f = \sum_{j,l \in \mathbb{Z}, k \in \mathbb{Z}^2} \langle f, \psi_{j,l,k} \rangle \psi_{j,l,k} \tag{5}$$

We use this formula to define the Shearlet Network approach, similar to the wavelet network [17], as a combination of the RBF neural network and the shearlet decomposition. In the optimization stage, a shearlet coefficient from the library is processed through the hidden layer of the network or used to update the weights. The calculation of the weights connection in every stage is obtained by projecting the signal to be analyzed on a family of shearlets. In our approach, the mother shearlet used to construct the family $\{\psi_{j,l,k}\}$ is the second derived of the Beta function [18].

Algorithm 1: SN learning

Input: image f

Output: reconstructed image f_{rec}

1. Select a shearlet $\{\psi_{j,l,k}\}$ as activation function of the shearlet network:

- Choose the mother shearlet.
- Build a library formed by the shearlets which form a shearlet frame.
- Set as a stop-learning condition (number of shearlets) and iterate the following steps:

2. Calculate the weights by direct projection of the image on the shearlet: $w_i = < f, \psi_{j,l,k} >$.

3. Calculate the output of the network f_{rec}.

4. Stop if the number of shearlets reaches the stop-learning condition, otherwise add another shearlet and return to **2.**

C. SMRSN Algorithm

We indicated in Sec. III.A. that the problem given by formula (3) is non-convex and can be relaxed using a multi-stage convex optimization as in formulation (4).

We recall that many successful optimization methods have been proposed in the literature, including the popular Lasso [19] and the iteratively reweighted least square [20], which achieves a sparse solution as in [2][3].

In our SMRSN approach, we adopt the recursive least square algorithm (RLS) [21] and the initial value of the weight w_{init} is chosen using the logistic function [22]:

$$w_{init} = 1/(1 + 1/\exp(-\mu e_{init}{}^2 + \mu\delta)) \tag{6}$$

where μ and δ are positive scalars and the initial residual e_{init} is given by:

$$e_{init} = (y - mean(X))^2 ; \tag{7}$$

here X is the aligned gallery of faces (an $n \times d$ matrix) and y a normalized test face (an $n \times 1$ matrix). Note that, after optimization by RLS, we can update the residual e and then the weight w_i.

Regarding the choice of the parameter $\lambda_j^{(\ell)}$, we have adopted the formula in [6]:

$$\lambda_j^{(\ell)} = \lambda I(|w_j^{(\ell)}| \le \theta) \tag{8}$$

where $j = 1, ..., d$ and

$$\lambda = \tau\sigma\sqrt{\ln(d)/n} \quad ; \quad \theta = \mu\lambda \tag{9}$$

with $\tau = 1, 2, 4, 8, 16, ...$ and $\mu = 0.5, 1, 2, ...$

Below we present the pseudocode of our SMRSN algorithm, where X represents the reconstructed gallery of faces after extraction of the features by training SN and y is the reconstructed test face with the features extracted after projection of the real test face on the frame of shearlets produced by the gallery faces.

Algorithm 2: SMRSN

Input: - y **:** normalized test face f : $y = f / norm(f, 2)$

- X : aligned gallery faces: $X = X / \sqrt{\sum X * X}$

- Iter: max of iteration

Output: w; $Identity(y)$

1. Compute the residual e_{init}, refer to formula (7)

2. Compute w_{init}, refer to formula (6)

3. Initialize $\lambda_j^{(0)} = \lambda$ and $\theta = \mu\lambda_j^{(0)}$ (as in formula (9))

4. For $\ell = 1,2,\ldots$

 For j = 1,…,Iter

 - Compute the formula (4) using RLS:

$$w_i = RLS(X, y, w_{init}, \lambda_j^{(\ell-1)}, \theta)$$

 - Update the residual $e : e = (Xw_i - y)^2$
 - $e_{init} = e$ (used for w_{init})
 - Update w_{init}, refer to formula (6)

 End

 - Update $\lambda_j^{(\ell)}$ and θ (as in formulas (8) -(9))
 - $w_{init} = w_i$

 End

 - $y_{rec} = X * w_i$
 - $w = w_i$

5. For k = 1,…, Classnum

$$error(k) = \left\| w^{1/2}(y - X_k w_k) \right\|_2^2$$

 End

6. $Identity(y) = \arg\min(error)$

Above, Classnum denotes the classes' number of X where $Classnum \geq d$; if $Classnum = d$ then we have the situation of single sample per person (SSPP).

The condition $w_{init} = w_i$ is one of the novelties of our SMRSN algorithm. In fact, for each new value of $\lambda_j^{(\ell)}$, RLS will use the term w_i from the previous stage (previous $\lambda_j^{(\ell)}$), a common assumption in MTL.

In the next section will describe the experimental results using our SMRSN approach.

IV. EXPERIMENTAL RESULTS

We have used the ORL, AR [23], Lab2 [24], FERET [25] and FRGC version 1 [32] databases in controlled environments and the LFW database [26] in uncontrolled environments to test the FR performance of the proposed method. We have used the Lab2 database for controlled different illumination environments and FERET, FRGC v1 databases for the case of single sample per person SSPP, one of the most challenging problem in FR.

We have used for comparison the SVM and NN methods, and several state-of-the-art FR methods including BHDT [27], MetaFace [28], RKR [29], RRC [3] and CRC [30]. For all these methods, we have used the codes provided by the authors with no change. All the face images were resized to 32×27.

A. AR and ORL Databases

In this set of experiments, we have selected a subset of 50 males and 50 females with only illumination and expression changes from the AR dataset [23]. For each subject, seven images from Session 1 were used for training, while other seven images from Session 2 were used for testing. The face images were resized to 32×27.

The ORL database contains 10 different images of each of 40 distinct subjects (400 images). For some subjects, the images were taken at different times, varying the lighting, facial expressions and facial details (glasses/no glasses). For each subject, we have randomly selected five images for training and other five images for testing. The face images were resized to 32×27.

The recognition accuracy on the AR and ORL database is shown in Table I. Our SMRSN method shows to a significant improvement in FR rate compared with the other methods considered.

TABLE I. RECOGNITION ACCURACY ON THE AR & ORL DATABASE.

Method	*AR*	*ORL*
NN	0.7010	-
SVM	0.8729	0.8700
BHDT [27]	0.5714	0.8000
MetaFace [28]	0.8814	0.8350
RKR [29]	0.9329	0.8100
RRC [3]	0.9257	0.8850
CRC [30]	0.9071	0.9000
SMRSN	**0.9500**	**0.9250**

B. Lab2 Database

The Lab2 database [24] contains visible light images and near-infrared images of the subjects. There are 50 subjects. Each subject provides twenty visible light face images (1000 images) and the same number of near-infrared face images. These images were acquired under four different illumination conditions (4 sessions). The face images also have variation in facial expression and pose. From the set of near-infrared faces, we have randomly chosen 5~15 samples from the first three sessions for training and 5 additional samples from the fourth session for testing. The face images were resized to 32×27.

Fig. 4. Two subjects from Lab2 database [24].

The recognition accuracy on the Lab2 database is shown in Table II. The proposed method shows superior performance with respect to all the other methods considered using 5 images and with RKR using 15 images, while the RRC achieves the best accuracy with 10 images and SMRSN achieves the second best results.

TABLE II. RECOGNITION ACCURACY ON THE LAB 2 DATABASE

Method	***5***	***10***	***15***
NN	-	-	-
SVM	0.6880	0.7880	0.8480
BHDT [27]	0.5880	0.7360	0.8200
MetaFace [28]	0.7320	0.7920	0.7600
RKR [29]	0.7200	0.8000	**0.8640**
RRC [3]	0.7360	**0.8320**	0.8440
CRC [30]	0.6680	0.8040	0.8480
SMRSN	**0.7400**	0.8080	**0.8640**

C. LFW Database

The LFW database [26] contains images of 5,749 different individuals in unconstrained environment. LFW-a is a version of LFW after alignment using commercial face alignment software [31]. We have extracted a dataset with 158 subjects from LFW-a. For each subject, we have randomly chosen 2~5 samples for training and another 2 samples for testing. The images are firstly cropped to 121×121 and then resized to 32×32 [34]. The FR rates on the LFW dataset are listed in Table III. The table shows that our SMRSN approach outperforms the other methods for two tests (3 and 5 images) while RKR give the best result with the others tests.

TABLE III. RECOGNITION ACCURACY ON THE LFW DATABASE

Method	***2***	***3***	***4***	***5***
NN	0,1100	0.1320	0.1470	0.1620
SVM	0.2152	0.2468	0.3038	0.3544
BHDT [27]	0.0791	0.1203	0.1361	0.1772
MetaFace [28]	0.1582	0.2152	0.2405	0.2563
RKR [29]	**0.3038**	0.3607	**0.4113**	0.4525
RRC [3]	0.2690	0.3449	0.3956	0.4462
CRC [30]	0.1899	0.2595	0.3322	0.3607
SMRSN	0.2785	**0.3681**	0.3734	**0.4589**

D. FERET and FRGC v1 Databases: SSPP

The FERET dataset contains a large number of subjects (single image per subject) in the gallery and the probe sets exploit differences in illumination, facial expression variations, and aging effects [25]. The frontal faces in the FERET database are divided into five sets: fa (1196 images, used as gallery set containing one image per person), fb (1195 images, taken with different expressions), fc (194 images, taken under different lighting conditions), dup1 (722 images, taken at a later date), and dup2 (234 images, taken at least one year apart). To test the SSPP problem, we have randomly chosen 100~200 images from fa for training and similarly 100~200 images from fb. The FR rates on the FERET dataset are listed in Table IV and show that our SMRSN algorithm achieves the best recognition accuracy compared to the others methods considered.

TABLE IV. RECOGNITION ACCURACY ON THE FERET DATABASE

Method	***100***	***150***	***200***
SVM	0.7700	0.7333	0.7150
BHDT [27]	0.5000	0.4200	0.3350
MetaFace [28]	0.8900	0.8933	0.8950
RKR [29]	0.8900	0.8533	0.8500
RRC [3]	0.8800	0.8800	0.9050
CRC [30]	0.8700	0.8400	0.8750
SMRSN	**0.9600**	**0.9400**	**0.9550**

FRGC v1 contains faces acquired under uncontrolled conditions [32]. We have used this dataset to test the challenging SSPP problem. Experiment 1 contains a single controlled gallery image and a probe with one controlled still image per subject (183 training images, 152 gallery images, and 608 probe images). Experiment 2 considers identification of a person given a gallery with four controlled still images per subject (732 training images, 608 gallery images, and 2432 probe images). Finally, experiment 3 considers a gallery with one controlled still image per subject and multiple uncontrolled probe images per subject (366 training images, 152 gallery images, and 608 probe images) [33]. We have randomly selected 152 images for training and 152 images for testing. The images were cropped and resized to 27×18 for the first experiment and resized to 32×27 for the second.

The recognition accuracy on the FRGC v1 database with two image sizes is shown in Table V. The proposed method shows superior performance with respect to all the other methods considered.

TABLE V. RECOGNITION ACCURACY ON THE FRGC V1 DATABASE

Method	***27×18***	***32×27***
NN	-	-
SVM	0.6053	0.6974
BHDT [27]	0.2697	0.2829
MetaFace [28]	0.6842	0.7171
RKR [29]	0.6316	0.6316
RRC [3]	0.7303	0.7697
CRC [30]	0.6513	0.7039
SMRSN	**0.7763**	**0.8158**

V. CONCLUSION

This paper has presented a novel high-performing face recognition method called Sparse Multi-Regularized Shearlet Network (SMRSN). One main novelty of our approach is that sparsity is achieved through the use of the shearlet representation, a method that combines multiscale analysis and directional selectivity. We have uses multi-task learning to learn features and ensure that the recognition problem is convex. Our approach is further refined by the recursive least square method (RLS) in the optimization step. Our experimental results on controlled and uncontrolled face databases show that our SMRSN algorithm is very competitive and outperforms state-of-the-art methods for face recognition, including the single sample per person situation.

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