**HW 8**

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

1. Let $f : D \to \mathbb{R}$ be continuous at $c \in D$. Prove that there exists an $M > 0$ and a neighborhood $U$ of $c$ such that $|f(x)| \leq M$ for all $x \in U \cap D$.

2. Determine the following limit
   \[ \lim_{x \to 0^-} \frac{4x}{|x|} \]
   (a) using the sequential definition;
   (b) using the $\epsilon - \delta$ definition.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be given by
   \[ f(x) = \begin{cases} 
   \sin(1/x) & \text{if } x \neq 0 \\
   0 & \text{if } x = 0.
   \end{cases} \]
   (a) Show that $f$ is not continuous at $x = 0$.
   (b) Show that $f$ has the intermediate property on any interval $[a, b] \in \mathbb{R}$, that is, if $k$ is any value between $f(a)$ and $f(b)$, then there exists $c \in (a, b)$ such that $f(c) = k$.

4. Show that any polynomial $p$ of odd degree has at least one real root.

5. Let $f : [a, b] \to [a, b]$ be continuous. Prove that $f$ must have a fixed point, that is, there is $c \in [a, b]$ such that $f(c) = c$. [Hint: Set $h(x) = f(x) - x$ and apply the Intermediate Value Theorem.]