**Quiz 6**

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1) [4pts] Find an example of a sequence \((s_n)\) of real numbers satisfying each set of properties.

(a) Cauchy, but not monotone.
\[ s_n = (-1)^n \frac{1}{n} \]

(b) Monotone, but not Cauchy.
\[ s_n = n \]

(c) Bounded, but not Cauchy.
\[ s_n = (-1)^n \]

(d) \((s_n)\) converges to 0 but \(\sum s_n\) is not a convergent series.
\[ s_n = \frac{1}{n} \]

(2) [4pts] Let \((a_n)\) be a sequence of nonnegative real numbers. Prove that \(\sum a_n\) converges iff the sequence of partial sums is bounded.

Proof. Since \(a_n \geq 0\) for all \(n\), then the sequence of partial sums \((s_n) = (\sum_{k \leq n} a_k)\) is monotone nondecreasing. It follows by the Monotone Convergence Theorem of sequences that \((s_n)\) converges iff it is bounded.