HW #1

Please, write clearly and justify all your steps, to get proper credit for your work.

(1) In a class of 125 students, the instructor gave 40 A’s, 30 B’s, 35 C’s, 15 D’s and 5 F’s. Compute the relative frequency of the 5 events. Compute the relative frequency of getting a grade that is better than C.

Solution.

\[ P(A) = \frac{40}{125} = \frac{8}{25}, \quad P(B) = \frac{30}{125} = \frac{6}{25}, \quad P(C) = \frac{35}{125} = \frac{7}{25}, \]
\[ P(D) = \frac{15}{125} = \frac{3}{25}, \quad P(F) = \frac{5}{125} = \frac{1}{25}. \]

\[ P(\text{better than C}) = P(B) + P(A) = \frac{14}{25}. \]

(2) Let \( S = A \cup B \), \( P(A) = 0.6 \), \( P(B) = 0.8 \).

(a) Find \( P(A \cap B) \)

(b) Find \( P(A^c \cup B^c) \)

Solution.

(a) \[ 1 = P(S) = P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
Hence \[ P(A \cap B) = P(A) + P(B) - P(A \cup B) = 1.4 - 1 = 0.4 \]
(b) \[ P(A^c \cup B^c) = P((A \cap B)^c) = 1 - P(A \cap B) = 0.6 \]

(3) Let \( P(A \cap B) = 0.2 \), \( P(A) = 0.5 \), \( P(B) = 0.4 \).

(a) Find \( P(A \cup B) \)

(b) Find \( P(A^c \cup B^c) \)

(c) Find \( P(A^c \cap B) \)

Solution.

(a) \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 - 0.2 = 0.7 \]
(b) \[ P(A^c \cup B^c) = P((A \cap B)^c) = 1 - P(A \cap B) = 1 - 0.2 = 0.8 \]
(c) \[ P(A^c \cap B) = P(B \setminus A) = P(B) - P(A \cap B) = 0.4 - 0.2 = 0.2 \]
(4) How many different ways can you rearrange the letters of the following words

(a) OLSEN
(b) CALCUTTA

Solution.

(a) The 5 letters are distinct. Number of permutations is:

\[(5)(4)(3)(2)(1) = 5!\]

(b) Of the 8 letters, the letters A, C and T are repeated. Number of permutations is:

\[\frac{8!}{2! 2! 2!}\]

(5) A bowl contains 20 chips, of which 9 are red, 8 are blue and 3 are white. Six chips are drawn at random and without replacement.

(i) Compute the probability that each of the 6 chips is red.

Solution.

\[P(6 \text{ red}) = \frac{\binom{9}{6}}{\binom{20}{6}}\]

(ii) Compute the probability that 3 chips are red and 3 chips are blue.

Solution.

\[P(3 \text{ red, 3 blue}) = \frac{\binom{9}{3} \times \binom{8}{3}}{\binom{20}{6}}\]