Math 3339 Name:

<u>HW #3</u>

Please, write clearly and justify all your steps, to get proper credit for your work.

(1)[8 Pts] Suppose that the probability density function $f(x)$ of the length X of an international phone call, rounded up to the next minute is given by:

x	1	2	3	4
$f(x)$	0.2	0.5	0.2	0.1

(a) Calculate $P(X \leq 2), P(X < 2)$, and $P(X \geq 1)$.
(b) Plot the cumulative distribution function $F(x)$.
(c) Calculate the mean $\mu = E(X)$.
(d) Calculate $E(X^2)$ and us it to compute the variance σ^2.

(a) $P(X \leq 2) = f(1) + f(2) = 0.2 + 0.5 = 0.7$; $P(X < 2) = f(1) = 0.2$; $P(X \geq 1) = \sum_{x=1}^{4} f(x) = 1$.
(c) $\mu = E(X) = \sum_{x=1}^{4} x\, f(x) = 1 \cdot 0.2 + 2 \cdot 0.5 + 3 \cdot 0.2 + 4 \cdot 0.1 = 2.2$
(d) $E(X^2) = \sum_{x=1}^{4} x^2\, f(x) = 1 \cdot 0.2 + 4 \cdot 0.5 + 9 \cdot 0.2 + 16 \cdot 0.1 = 5.6$. $\sigma^2 = E(X^2) - \mu^2 = 5.6 - (2.2)^2 = 0.76$

(2)[8 Pts] A job applicant to a company is required to submit one, two, three, four, or five forms depending on the nature of the job. Let X to denote the number of forms required of an applicant. The probability that x forms are required is known to be proportional to x, that is,

$$p(x) = k\, x, \text{ for } x = 1, 2, \ldots, 5.$$

(a) Calculate the value k so that $p(x)$ is a probability mass function.
(b) What is the probability that at least 2 forms are needed?
(c) What is the probability that at most 2 forms are needed?
(d) Calculate $E(X^2)$ and us it to compute the variance σ^2.

(a) $1 = \sum_{x=1}^{5} k\,x = k\,(1+2+3+4+5) = 15\,k$. Hence $k = \frac{1}{15}$.
(b) $P(X \geq 2) = 1 - P(X < 2) = 1 - f(1) = \frac{14}{15}$.
(c) $P(X \leq 2) = f(1) + f(2) = \frac{3}{15}$.
(d) $E(X^2) = \sum_{x=1}^{5} x^2\,f(x) = \frac{1}{15}\sum_{x=1}^{5} x^3 = 15$. $\mu = E(X) = \sum_{x=1}^{5} x\,f(x) = \frac{1}{15}\sum_{x=1}^{5} x^2 = \frac{11}{3}$. $\sigma^2 = E(X^2) - \mu^2 = 15 - (\frac{11}{3})^2 = \frac{14}{9}$.

(3)[10 Pts] This problem requires R. Using the data of Problem (1)

(a) Plot the probability mass function. Remember to label the x and y axes.
(b) Verify that the values of the probability add up to 1.
(c) Plot the cumulative distribution function. Remember to label the x and y axes.

You need to print your plots and your code.

(a)

```
x <- c(i=1:4)
y <- c(0.2,0.5,0.2,0.1)
plot(x,y, xlab="x", ylab="pmf", main="probability mass function",
ylim=c(0,1), xlim=c(0,5), pch=15, col="blue")
```

Alternatively:

```
plot(x,y, type="h", xlab="x", ylab="pmf", main="probability mass
function", ylim=c(0,1), xlim=c(0,5), lwd=2,col="black")
points(x,y,pch=16,cex=2,col="blue")
```

(b)

```
sum(p)
```

(c)

```
x <- c(i=1:4)
y2 <- c(0.2,0.7,0.9,1)
plot(x,y2, type="s",xlab="x", ylab="cdf", main="cumulative density
function", ylim=c(0,1), xlim=c(0,5), pch=15, col="blue")
```