Math 3339 Name: SOLUTION

HW #4

Please, write clearly and justify all your steps, to get proper credit for your work.

(1)[3 Pts] The probability of producing a high-quality color print is 0.10. How many prints do you have to produce so that the probability of producing at least one quality print is larger than 0.90?

P(at least 1 quality print) = 1 - P(no quality print, n attempts) $= 1 - 0.9^n > 0.9$.

Hence

$$1 - 0.9 > 0.9^n$$
$$0.1 > 0.9^n$$
$$\log 0.1 > n \log 0.9$$
$$n > \frac{\log 0.1}{\log 0.9} = 21.8.$$

You can round up to $n = 22$.

Alternatively, we can examine the values of $P(X \geq 1)$ *where* $X \sim bin(n, p = 0.1)$ *until we find n for which the probability value is larger than 0.9. Note that* $P(X \geq 1) = 1 - P(X = 0)$

Using R we find:

```
> vec<-c()
> for (n in 1:100)
+ vec[n] <- 1-dbinom(0,n,0.1)
> min(which(vec>0.9))
[1] 22
```

(2)[5 Pts] The university football team has 11 games on its schedule. Assume that the probability of winning each game is 0.40 and that there are no ties. Assuming independence, what is the probability that this year's team will have a winning season, that is, that the team will win at least six games?

Set $Y \sim b(n = 11, p = 0.4)$. *Hence*

$$P(Y \geq 6) = 1 - P(Y \leq 5) = 1 - F(5) = 1 - 0.7535 = 0.2465.$$

Using R:

```
> 1-pbinom(5,11,0.4)
[1] 0.2465019
```

or

```
> pbinom(5,11,0.4,lower=FALSE)
[1] 0.2465019
```

(3)[4 Pts] If a student answers questions on a true-false test randomly (i.e., assume that $p = 0.5$) and independently, determine the probability that:

(a) the first correct answer is in response to question 4;

(b) at most four questions (that is, four or fewer) must be answered to get the first correct answer.

Set $Z \sim nb(r = 1, p = 0.5)$. *Hence*

$$(a)\ \ P(Z = 4) = (0.5)\,(0.5)^3 = (0.5)^4 = 0.0625.$$

Using R (in R the negative binomial counts the number of failures before the first success):

```
> dnbinom(3,1,0.5)
[1] 0.0625
```

$$(b)\ \ P(Z \le 4) = \sum_{z=1}^{4} (0.5)\,(0.5)^{z-1} = (0.5)^1 + (0.5)^2 + (0.5)^3 + (0.5)^4 = 0.9375.$$

Using R:

```
> pnbinom(3,1,0.5)
[1] 0.9375
```

(4)[3 Pts] Suppose that there are 100 defective items in a lot of 2000 items. If a sample of size 10 is taken at random and without replacement, what is the probability that there are two or fewer defectives in the sample?

Set $W \sim hypergeom(N_1 = 100, N_2 = 1900, n = 10)$. *Hence*

$$P(W \le 2) = \sum_{k=0}^{2} \frac{\binom{100}{k}\binom{1900}{10-k}}{\binom{2000}{10}} = 0.9887.$$

Using R:

```
> phyper(2,100,1900,10)
[1] 0.9887401
```

Since N, N_1 are much larger than n, we can approximate solution with the binomial pmf where $W \sim b(n = 10, p = \frac{100}{2000} = 0.05)$ gives

$$P(W \leq 2) \approx F(2) = 0.9885.$$

Using R:

```
> pbinom(2,10,0.05)
[1] 0.9884964
```

(5)[6 Pts] A stockbroker has a 60 percent probability of success in picking stocks that appreciate. Assume independence. You are investing in 20 securities that he suggested. Calculate the probability that

(a) 9, 10 or 11 stocks will appreciate;

(b) that fewer than 14 stocks will appreciate;

(c) calculate the mean and standard deviation of the number of the stocks that will appreciate.

Set $Y \sim b(n = 20, p = 0.60)$. *Hence*
(a) $P(9 \leq Y \leq 11) = F(11) - F(8) = 0.4044 - 0.0565 = 0.3479.$
Using R:

```
> pbinom(11,20,0.6)-pbinom(8,20,0.6)
[1] 0.3478749
```

(b) $P(Y < 14) = P(Y \leq 13) = F(13) = 0.7500.$
Using R:

```
> pbinom(13,20,0.6)
[1] 0.7499893
```

(c) $\mu = n\,p = 20\,0.60 = 12$; $\sigma = \sqrt{np(1-p)} = \sqrt{20\,0.60\,0.40} = 2.19.$