Math 3339
Name: SOLUTION

HW #5

You have the following commands in R to compute probabilities associated with Poisson distributions.
- `dpois(x, lambda)`: \( P(X = x) \) for \( X \sim \text{Poisson}(\lambda) \)
- `ppois(q, lambda)`: \( P(X \leq q) \) for \( X \sim \text{Poisson}(\lambda) \)

(1)[3 Pts] On average, 2.5 telephone calls per minute are received at the UH’s switchboard. Assuming that the number of incoming calls per minute follows a Poisson distribution, compute the probability that at any given minute there will be more than 2 calls.

Denote as \( X \) the number of incoming calls per minute. Thus
\[
P(X > 2) = 1 - P(X \leq 2)
\]

Using R with \( \lambda = 2.5 \):
```r
> 1 - ppois(2, 2.5)
[1] 0.4561869
```

(2)[3 Pts] Suppose that in one year the number of industrial accidents \( X \) follows a Poisson distribution with mean 3.0. If each accident leads to an insurance claim of $5,000, how much money would an insurance company need to keep in reserve to be 95% certain that the claims are covered?

You can list the values of the cumulative Poisson distribution with \( \lambda = 3 \) until you find a value above 0.95. Using R:
```r
> ppois(4, lambda=3)
[1] 0.8152632
> ppois(5, lambda=3)
[1] 0.9160821
> ppois(6, lambda=3)
[1] 0.9664915
```

Hence to be 95% confident to be covered, the insurance company should be expected to cover up to 6 claims per year. Thus, it needs to set aside $6 \cdot 5,000 = $30,000.

(3)[4 Pts] A delivery company found that the number of complaints was six per years on average. Assuming that the number of complaints follows a Poisson distribution, calculate the probability of having no complaints in

(a) all of next year;
(b) the next quarter.

(a) Poisson distribution with \( \lambda = 6 \). We compute \( P(X = 0) \):
```r
> dpois(0, lambda=6)
[1] 0.002478752
```

(b) Poisson distribution with \( \lambda = 1.5 \). \( P(X = 0) \):
```r
> dpois(0, lambda=1.5)
[1] 0.2231302
```

(4)[6 Pts] Let \( X \) and \( Y \) have the following joint p.d.f.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
<td>0.15</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

(a) Calculate the marginal densities. Are \( X \) and \( Y \) are independent?

(b) Compute the means and variances.

(c) Are \( X \) and \( Y \) positively correlated? negatively correlated? uncorrelated?

Here is the solution with the R.

```r
> p <- matrix(c(.05,.10,.15,.15,.10,.15,.15,.10,.05),ncol=3)
> px <- apply(p,2,sum)  ## column-sum: it creates marginal probabilities for \( X \)
> px
[1] 0.3 0.4 0.3
> py <- apply(p,1,sum)  ## row-sum: it creates marginal probabilities for \( Y \)
> py
[1] 0.35 0.30 0.35
> x <- c(1,2,3)
> y <- c(1,2,3)
> EX <- sum(px*x)
> EX
[1] 2
> EY <- sum(py*y)
> EY
[1] 2
> EX2 <- sum(px*x*x)
> EY2 <- sum(py*y*y)
> VarX <- EX2-EX*EX
> VarX
[1] 0.6
> VarY <- EY2-EY*EY
> VarY
[1] 0.7
> A=0
> for(i in 1:3)for(j in 1:3)A <- A+p[i,j]*x[i]*y[j]
> EXY<-A
> EXY
[1] 3.8
> COVXY <- EXY-EX*EY
> COVXY
[1] -0.2
Negative correlation.
\( X, Y \) not independent since \( f(2,2) = 0.10 \neq f_1(2) \ast f_2(2) = 0.4 \ast 0.3 \)
(5)[4 Pts] Let $W = 1 - X + 2Y$ be a discrete random variable where $X$, $Y$ are independent discrete random variables with $\mu_X = 5$, $\mu_Y = 2$, and $\sigma_X^2 = 1$, $\sigma_Y^2 = 2$. Compute $\mu_W$ and $\sigma_W^2$.

$\mu_W = 1 - \mu_X + 2\mu_Y = 1 - 5 + (2)(2) = 0$

$\sigma_W^2 = \sigma_X^2 + 4\sigma_Y^2 = 1 + (4)(2) = 9$