To find the numerical solutions, you can use the statistical tables or the commands `pnorm` and `qnorm` in R.

(1)[4 Pts] Let $\bar{X}$ be the mean of a random sample of size $n = 48$ from the uniform distribution in the interval $(0, 2)$. Approximate the probability $P(0.9 < \bar{X} < 1.1)$ using the Central Limit Theorem.

By the properties of the uniform distribution, $\mu = 1$, $\sigma^2 = \frac{(0-2)^2}{12} = \frac{1}{3}$

Hence $\mu_\bar{x} = 1$, $\sigma^2_\bar{x} = \frac{1}{3 \cdot 48} = \frac{1}{144}$, $\sigma_\bar{x} = 1/12$

$$P(0.9 < \bar{X} < 1.1) = \text{pnorm}(1.1, 1, 1/12) - \text{pnorm}(0.9, 1, 1/12) = 0.7698$$

(2)[4 Pts] Let $\bar{X}$ be the mean of a random sample of size $n = 48$ from a distribution with mean 4 and variance 16. Approximate the probability $P(3.1 < \bar{X} < 4.6)$ using the Central Limit Theorem.

Hence $\mu_\bar{x} = 4$, $\sigma^2_\bar{x} = \frac{16}{48}$, $\sigma_\bar{x} = 4/\sqrt{48}$

$$P(3.1 < \bar{X} < 4.6) = \text{pnorm}(4.6, 4, 4/\sqrt{48}) - \text{pnorm}(3.1, 4, 4/\sqrt{48}) = 0.792$$

(3)[4 Pts] The profits from investments in individual stocks follow a normal distribution with mean 1 and standard deviation 5.

(a) If are buying a single random selected stock, what is the probability that your profit is greater than zero?

(b) If are buying a portfolio of 25 randomly selected stocks, what is the probability that your average profit is greater than zero?

$$X \sim N(\mu = 1, \sigma^2 = 25)$$

(a) $n = 1$, $\mu_\bar{X} = 1$, $\sigma^2_\bar{X} = \frac{25}{1}$, $\sigma_\bar{X} = 5$

$$P(\bar{X} > 0) = 1 - P(\bar{X} \leq 0) = 1 - \text{pnorm}(0, 1, 5) = 0.5793$$
(b) \( n = 25, \mu_X = 1, \sigma^2_X = \frac{25}{25}, \sigma_X = 1 \)

\[
P(\bar{X} > 0) = 1 - P(\bar{X} \leq 0) = 1 - \text{pnorm}(0, 1, 1) = 0.8413
\]

(4)[4 Pts] The mean and standard deviation measured from a randomly selected sample of \( n = 42 \) mathematics SAT test scores are \( \bar{x} = 680 \) and \( s = 35 \). Find an approximate 99 percent confidence interval for the population mean \( \mu \).

99% confidence interval. \( 1 - \alpha = 0.99 \Rightarrow \alpha = 0.01 \Rightarrow z_{\alpha/2} = z_{0.005} = 2.576 \)

\[
[\bar{X} - z_{\alpha/2} \frac{s}{\sqrt{n}}, \bar{X} + z_{\alpha/2} \frac{s}{\sqrt{n}}] = [680 - 2.576 \frac{35}{\sqrt{42}}, 680 + 2.576 \frac{35}{\sqrt{42}}] = [666.1, 693.1]
\]

(5)[4 Pts] Let a population be normally distributed with mean \( \mu \) and standard deviation \( \sigma = 5 \). Find the sample size \( n \) such that we are 95 percent confident that the estimate of \( \bar{x} \) is within \( \pm 1.5 \) unit of the true mean \( \mu \).

95% confidence interval. \( 1 - \alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow z_{\alpha/2} = z_{0.025} = 1.96 \)

\[
n \geq z_{0.025}^2 \frac{\sigma^2}{n} = 1.96^2 \frac{5^2}{1.5^2} = 42.68
\]

Choose \( n = 43 \)

(6)[4 Pts] The EPA considers indoor radon levels above 4 picocuries per liter (pCi/L) of air to be high enough to warrant amelioration efforts. Tests in a sample of 200 homes in a certain county found 127 (63.5\%) of these sampled households to have indoor radon levels above 4 pCi/L. Compute the 95% confidence interval of the proportion of all the households in the county that don’t meet the EPA guidelines.

95% confidence interval. \( 1 - \alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow z_{\alpha/2} = z_{0.025} = 1.96 \)

\[
[\bar{X} - z_{\alpha/2} \frac{1}{2\sqrt{n}}, \bar{X} + z_{\alpha/2} \frac{1}{2\sqrt{n}}] = [0.635 - \frac{1.960}{2\sqrt{200}}, 0.635 + \frac{1.960}{2\sqrt{200}}] = [0.566, 0.704]
\]