Math 3339 Name:

HW #9

Please, write clearly and justify all your steps, to get proper credit for your work.

(1)[5 Pts] Let μ be the mileage of a certain brand of tire. A sample of $n = 22$ tires is taken at random, resulting in the sample mean $\overline{x} = 29,132$ and sample variance $s^2 = 2,236$. Assuming that the distribution is normal, find a 99 percent confidence interval for μ.

$$CI: \ 29132 \pm t(0.005; 21)\frac{\sqrt{2236}}{\sqrt{22}} = 29132 \pm 2.831\frac{\sqrt{2236}}{\sqrt{22}} = [29103.4, 29160.6]$$

(2)[5 Pts] We need to estimate the average of a normal population and from measurements on similar populations we estimate that the sample mean is $s^2 = 9$. Find the sample size n such that we are 90 percent confident that the estimate of $\overline{x}$ is within ± 1 unit of the true mean μ.

$$n \geq z_{0.05}^2 \frac{s^2}{h^2} = 1.645^2 \cdot 9 = 24.4 \Rightarrow n = 25$$

(3)[5 Pts] In comparing the times until failure (in hours) of two different types of light bulbs, we obtain the sample characteristics $n_1 = 45$, $\overline{x} = 984$, $s_x^2 = 8,742$ and $n_2 = 52$, $\overline{y} = 1,121$, $s_y^2 = 9,411$. Find an approximate 90% confidence interval for the difference of the two population means.

$$CI: \quad (984 - 1121) \pm 1.645\sqrt{\frac{8742}{45} + \frac{9411}{52}} = [-168.9, -105.1]$$

(4)[5 Pts] Two rubber compounds were tested for tensile strength and the following values were found

$$A: \ 32, 30, 33, 32, 29, 34, 32$$
$$B: \ 33, 35, 36, 37, 35$$

Assuming that the two populations are normally distributed and have the same variance, find a 95% confidence interval for the difference of the two population means.

From the data:

$$A : n_x = 7 \quad \bar{x} = 31.71, \quad s_x = 1.704$$

$$B : n_y = 5 \quad \bar{y} = 35.20, \quad s_y = 1.483$$

$$t(0.025; 10) = 2.228 \quad s_p^2 = \frac{6\,1.704^2 + 4\,1.483^3}{10} = 2.622$$

$$CI : \quad (31.71 - 35.20) \pm 2.228\sqrt{2.622}\sqrt{\frac{1}{7} + \frac{1}{5}} = [-5.60, -1.38]$$

(5)[5 Pts] Lightbulbs of a certain type are advertised as having an average lifetime of 750 hours. A random sample of 50 bulbs was selected, the lifetime of each bulb determined finding that the sample average lifetime is 738.5 with sample standard deviation 38.2. Test the hypothesis that the true average lifetime is smaller than what is advertised using significance level $\alpha = 0.05$ and $\alpha = 0.01$.

We test the hypothesis

$H_0 : \mu = 750$;

$H_1 : \mu < 750$.

Test statistic (z-test) $W = \frac{\bar{X}-\mu_0}{s/\sqrt{n}} = \frac{738.5-750}{38.2/\sqrt{50}} = -2.130$

For $\alpha = 0.05$, $z_{0.05} = 1.645$; *for* $\alpha = 0.01$, $z_{0.01} = 2.326$

Since, $W = -2.130 < -1.645$, *we reject* H_0 *at significance level* $\alpha = 0.05$.

Since, $W = -2.130 > -2.326$, *we fail to reject* H_0 *at significance level* $\alpha = 0.01$.

p-value: $P(z < -2.130) = 0.017$