

Quiz #5

Please, show your work and write legibly. Recall the following R commands:

`dpois(x, lambda)`:  $P(X = x)$  for  $X \sim \text{Poisson}(\lambda)$

`ppois(q, lambda)`:  $P(X \leq q)$  for  $X \sim \text{Poisson}(\lambda)$

(1)[4 Pts] A delivery company found that the number of complaints was 12 per years on average. Assuming that the number of complaints follows a Poisson distribution, calculate the probability of having

- (a) at most 8 complaints in all of next year;
- (b) 8 complaints or more in all of next year.

Let  $X \sim \text{pois}(12)$

(a)  $P(X \leq 8) = \text{ppois}(8, 12) = 0.1550278$ .

(b)  $P(X \geq 8) = 1 - P(X \leq 7) = 1 - \text{ppois}(7, 12) = 0.9104955$ .

(2) [6 Pts] Let  $X$  and  $Y$  have the following joint p.d.f.

	<b>x</b>		
<b>y</b>	1	2	3
1	0.10	0.15	0.15
2	0.05	0.10	0.10
3	0.10	0.20	0.05

- (a) Calculate the means with respect to  $X$  and  $Y$
- (b) Are  $X$  and  $Y$  dependent or independent? Justify our answer.
- (c) Are  $x$  and  $Y$  positively correlated? negatively correlated? uncorrelated? Justify your answer?

(a) By direct computation the marginal probabilities are  $f_1(x) = (0.25, 0.45, 0.30)$  and  $f_2(y) = (0.40, 0.25, 0.35)$

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> px <-c(0.25,0.45,0.30)
> py <-c(0.40,0.25,0.35)
> x <- c(1,2,3)
> y <- c(1,2,3)
> EX <- sum(px*x)
> EY <- sum(py*y)
> print(EX) = 2.05
> print(EY) = 1.95
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(b) Since  $f_1(1)f_2(1) \neq f(1, 1)$ , the  $X$  and  $Y$  are dependent.

(c)  $E[XY] = 1(0.1)+2(0.15)+3(0.15)+2(0.05)+4(0.1)+6(0.1)+3(0.1)+6(0.2)+9(0.05) = 3.9$   
 $\sigma_{xy} = E[XY] - \mu_x\mu_y = 3.9 - (1.95)(2.05) = -0.0975$ . Thus,  $X, Y$  are **NEGATIVELY** correlated.