QUIZ #6

Please, use R to compute the numerical solutions.

(1) [4 Pts] Let $\bar{X}$ be the mean of a random sample of size $n = 48$ from the uniform distribution in the interval (2, 8). Approximate the probability $P(4.9 < \bar{X} < 5.5)$ using the Central Limit Theorem. You need to show how you set up the probability calculation.

By the properties of the uniform distribution, $\mu = 5$, $\sigma^2 = \frac{(8-2)^2}{12} = 3$
Hence $\mu_{\bar{x}} = 5$, $\sigma^2_{\bar{x}} = \frac{3}{48} = \frac{1}{16}$ and

$$P(4.9 < \bar{X} < 5.5) = \text{pnorm}(5.5, 5, 1/4) - \text{pnorm}(4.9, 5, 1/4) = 0.9773 - 0.3446 = 0.6327$$

(2) [4 Pts] Let a population be normally distributed with mean $\mu$ and standard deviation $\sigma = 5$. Find the minimal sample size $n$ such that we are 99 percent confident that the estimate of $\bar{x}$ is within $\pm 1.2$ unit of the true mean $\mu$.
You need to show the formula you apply to find your numerical solution.

$$z_{0.005} = 2.576$$
$$n \geq z_{0.005}^2 \frac{\sigma^2}{\bar{x}^2} = 2.576^2 \frac{5^2}{1.2^2} = 115.20$$
Choose $n = 116$