(1)[6 Pts] Let $\bar{X}$ be the mean of a random sample of size $n = 48$ from the uniform distribution in the interval (2,8). Approximate the probability $P(4.9 < \bar{X} < 5.5)$ using the Central Limit Theorem. You must show how you set up the probability calculation.

By the properties of the uniform distribution, $\mu = \frac{8+2}{2} = 5$, $\sigma^2 = \frac{(8-2)^2}{12} = 3$
Hence $\mu_x = 5$, $\sigma^2_x = \frac{3}{48} = \frac{1}{16}$ and $\bar{X} \sim N(\mu = 5, \sigma = 1/4)$

$P(4.9 < \bar{X} < 5.5) = \text{pnorm}(5.5, 5, 1/4) - \text{pnorm}(4.9, 5, 1/4) = 0.977 - 0.3446 = 0.633$

(2)[4 Pts] Let a population be normally distributed with mean $\mu$ and standard deviation $\sigma = 5$. Find the minimal sample size $n$ such that we are 99 percent confident that the estimate of $\bar{x}$ is within $\pm 1.2$ unit of the true mean $\mu$. You must show the formula you apply to find your numerical solution.

$z_{0.005} = \text{qnorm}(1 - 0.005) = 2.576$
$n \geq z_{0.005}^2 \frac{\sigma^2}{n} = 2.576^2 \frac{5^2}{1.2^2} = 115.20$
We choose $n = 116$