(1) [3Pts] A rubber compound were tested for tensile strength and the following values were found:

32, 30, 31, 33, 32, 30, 35, 34, 32, 31

(a) Apply the Shapiro-Wilk test to verify that the data can be modeled according to a normal distribution.

We first apply the Shapiro-Wilk normality test.
> x <- c(32, 30, 31, 33, 32, 30, 35, 34, 32, 31)
> shapiro.test(x)

Shapiro-Wilk normality test

data: x
W = 0.93455, p-value = 0.4941

Since the p-value is larger than 0.05, we can assume that the data are normally distributed.

We test the hypothesis

\[ H_0 : \mu = 31; \]
\[ H_1 : \mu \neq 31. \]

Test statistic (t-test) \( W = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{32 - 31}{1.633/\sqrt{10}} = 1.9365 \)

For \( \alpha = 0.05 \), \( \alpha/2 = 0.025 \), \( t_{0.025,9} = 2.26216 \)

Since, \( W = 1.9365 < 2.26216 \), we fail to reject \( H_0 \) at significance level \( \alpha = 0.05 \).

p-value = \( 2P(t < -1.9365) = (\text{using R: } 2*pt(-1.9365,9)) = 0.08479 \)

R solution
> x <- c(32, 30, 31, 33, 32, 30, 35, 34, 32, 31)
> t.test(x,mu=31,alternative = "two.sided", conf.level = 0.95)

One Sample t-test
data: x
t = 1.9365, df = 9, p-value = 0.08479
alternative hypothesis: true mean is not equal to 31
sample estimates:
mean of x
32
(2) [2Pts] Using the same data as in Problem 4 and still assuming that the population is normally distributed, test the hypothesis that the average tensile strength is larger than 31. Use \( \alpha = 0.05 \). Calculate the \( p \)-value of the test.

We test the hypothesis

\[
H_0 : \mu = 31; \\
H_1 : \mu > 31.
\]

> x <-c(32, 30, 31, 33, 32, 30, 35, 34, 32, 31)
> t.test(x, mu=31, alternative = "greater", conf.level = 0.95)

One Sample t-test
data: x
t = 1.9365, df = 9, p-value = 0.04239alternative hypothesis: true mean is not equal to 31sample estimates:
mean of x 32

In this case, p-value is less than 0.05, hence we reject \( H_0 \) at significance level \( \alpha = 0.05 \).

(3) [3Pts] Two rubber compounds were tested for tensile strength and the following values were found

\[
A : \quad 32, 30, 33, 32, 29, 34, 32 \\
B : \quad 33, 35, 36, 37, 36
\]

Under the assumption that the two populations are normally distributed and have the same (unknown) variance, test the hypothesis that the average tensile strength of the two rubber compounds is different using significance level \( \alpha = 0.01 \) and \( \alpha = 0.005 \).

We test \( H_0 : \mu_A = \mu_B \) against \( H_1 : \mu_A \neq \mu_B \) with \( \alpha = 0.05 \) and \( \alpha = 0.01 \).

Since the populations are normal and the variance is unknown, we run a 2-population t-test

We first find the sample means \( \bar{x}_A = 31.714 \) and \( \bar{x}_B = 35.000 \) and the sample variances

\[
s_A^2 = 2.905 \quad \text{and} \quad s_B^2 = 2.800.
\]

We compute the pooled variance

\[
s_p^2 = \frac{(n_A-1)s_A^2 + (n_B-1)s_B^2}{n_A + n_B - 2} = 2.857
\]

Test statistic (Student t pdf):

\[
t = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{s_p^2/n_A + s_p^2/n_B}} = \frac{31.714 - 35.000}{\sqrt{2.857/7 + 2.857/6}} = -3.494
\]

Rejection regions: \( |t| > t_{0.025;11} = 2.201, \quad |t| > t_{0.005;11} = 3.496 \)

Since \( t < -t_{0.025;11} = -2.201 \) and \( t < -t_{0.005;11} = -3.496 \) then \( H_0 \) is REJECTED at significance level \( \alpha = 0.01 \) and \( H_0 \) is ACCEPTED at significance level \( \alpha = 0.005 \).

One can solve using R with
> t.test(x,y,alternative = "two.sided", paired = FALSE, var.equal = TRUE, conf.level = 0.95)
  You find: p-value = 0.005023
  Since the p-value is less than $\alpha = 0.01$ but larger than and $\alpha = 0.005$, then we reject $H_0$ at significance level $\alpha = 0.01$ but not at significance level $\alpha = 0.005$. 