SOLUTION of TEST #1

(1)

(i) Sample space contains $\binom{20}{5}$ events which are equally likely

$$P(5 \text{ blue clips}) = \boxed{\frac{\binom{7}{5}}{\binom{20}{5}}}$$

← You can choose 5 blue clips in $\binom{7}{5}$ ways

Answ: $\frac{7}{20} \cdot \frac{6}{19} \cdot \frac{5}{18} \cdot \frac{4}{17} \cdot \frac{3}{16}$

(ii) $$P(1 \text{ blue}, 3 \text{ white}, 1 \text{ red}) = \boxed{\frac{\binom{7}{1}\binom{8}{3}\binom{5}{1}}{\binom{20}{5}}}$$

(iii) $$P(\text{no blue clips}) = \boxed{\frac{\binom{13}{5}}{\binom{20}{5}}}$$

← You can choose 5 clips out of 13 possible ones

Answ: $\frac{13}{20} \cdot \frac{12}{19} \cdot \frac{11}{18} \cdot \frac{10}{17} \cdot \frac{9}{16}$

(iv) $$P(\text{at least one blue clip}) = 1 - P(\text{no blue clips}) = \boxed{1 - \frac{\binom{13}{5}}{\binom{20}{5}}}$$

(2)

(i) $$P(3 \text{ defective bulbs in 3 draws}) = \frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10} = \boxed{\frac{1}{220}}$$

(ii) $$P(5\text{th bulb}) = P\big(\underbrace{(2 \text{ defective bulbs in 4 draws})}_{A_1} \cap \underbrace{(\text{defective bulb in 5th draw})}_{A_2}\big) =$$

$$= P(A_1) \cdot P(A_2 | A_1) =$$

$P(A_2|A_1) = \frac{1}{8}$ since only one defective bulb is left among 8 bulbs

$$= \frac{\binom{3}{2}\binom{9}{2}}{\binom{12}{4}} \cdot \frac{1}{8} = \boxed{\frac{3}{110}}$$

(iii) $$P(10\text{th bulb}) = P\big(\underbrace{(2 \text{ defective bulbs in 9 draws})}_{B_1} \cap \underbrace{(\text{defective bulb in 10th draw})}_{B_2}\big)$$

$$= \frac{\binom{3}{2}\binom{9}{7}}{\binom{12}{9}} \cdot \frac{1}{3} = \boxed{\frac{9}{55}}$$

(3)

Denote by A_1, A_2, A_3 the events of East, Midwest, West, resp.
and by B_1, B_2 the events of Pessimistic, Optimistic, resp.

(i) 300 respondents out of 500 are Pessimistic. $\boxed{P(B_1) = \frac{3}{5}}$

(ii) 160 respondents are from Midwest $P(A_2) = \frac{16}{50} = \frac{8}{25}$

$$P(B_2 | A_2) = \frac{P(A_2 \cap B_2)}{P(A_2)} = \frac{70/500}{160/500} = \boxed{\frac{7}{16}}$$

(iii) The respondents who are optimistic are 200. $P(B_2) = \frac{200}{500}$

$$P(A_2 | B_2) = \frac{P(A_2 \cap B_2)}{P(B_2)} = \frac{70/500}{200/500} = \boxed{\frac{7}{20}}$$

(iv) $P(A_2 \cap B_2) = 70/500$ But $P(A_2)\, P(B_2) = \frac{160}{500} \cdot \frac{200}{500} = \frac{64}{500}$

Since $P(A_2 \cap B_2) \neq P(A_2) P(B_2)$, the events A_2, B_2 are NOT INDEP

Similar

$$P(A_3 \cap B_1) = \frac{110}{500} \neq P(A_3)\, P(B_1) = \frac{200}{500} \cdot \frac{300}{500} = \frac{120}{500}$$

WANT

$$P(A_3 \cap B_1) = P(A_3)\, P(B_1) = \boxed{\frac{120}{500}}$$

$$P(A_3 \cap B_2) = P(A_3)\, P(B_2) = \frac{200}{500} \cdot \frac{200}{500} = \boxed{\frac{80}{500}}$$

	A_3
B_1	120
B_2	80

(4) D: disease ; TP : test positive ; TN : test negative

We have $P(D) = 0.04$, $P(TN | D) = 0.20$, $P(TP | D') = 0.08$

$\Rightarrow$ We define $P(D') = 0.96$, $P(TP | D) = 0.80$ $\left(= 1 - P(TN|D)\right)$

Thus, by Baye's thm:

$$T(D | TP) = \frac{P(TP|D)\, P(D)}{P(TP|D)\, P(D) + P(TP|D')\, P(D')} = \frac{(0.80)(0.04)}{(0.80)(0.04) + (0.08)(0.96)}$$

$$= \frac{5}{17}$$