Math 3339 Name: SOLUTION

Test #2

(1)[6 Pts] An experimental medication was given to patients with a certain medical condition. Suppose that the probability that a patient with the underlying condition will experience severe side effects if given that medication is 8.5%. What is the probability that, of 35 randomly chosen such patients,

(a) more than 3 will experience severe side effects?

(b) exactly 3 will experience severe side effects?

(c) How many of the 35 patients do you expect will experience severe side effects?

Number of patients with severe side effects: $X \sim bin(n = 35, p = 0.085)$

(a) $P(X > 3) = 1 - P(X \leq 3)$ => `1- pbinom(3,35, 0.085)` $= \boxed{0.347}$

(b) $P(X = 3) =$ `dbinom(3,35, 0.085)` $= \boxed{0.234}$

(c) $E(X) = 35 * 0.085 = \boxed{2.975}$

(2)[8 Pts] Let X and Y have the following joint p.d.f.

y \ **x**	1	2
1	0.14	0.32
2	0.39	0.15

(a) Calculate the marginal densities. Are X and Y are independent?

(b) Compute the means and variances.

(c) Are X and Y positively correlated? negatively correlated? uncorrelated?

(d) Let $Z = 1 - X + 2Y$. Find the mean and the variance of Z.

(a)
```
> p <- matrix(c(.14,.39,.32,.15),ncol=2)
> px <- apply(p,2,sum)
[1] 0.53 0.47
> py <- apply(p,1,sum)
[1] 0.46 0.54
```
That is: $f_1(x) = \boxed{(0.53, 0.47)}$ and $f_2(y) = \boxed{(0.46, 0.54)}$

X, Y not independent since $f(1,1) = 0.14 \neq f_1(1) * f_2(1) = 0.53 * 0.46 = 0.244$

(b-c)
```
> x <- c(1,2)
> y <- c(1,2)
> EX = sum(px*x) = 1.47
> EY = sum(py*y) =1.54
> VarX = sum(px*x*x)-EX*EX = 0.249
> VarY = sum(py*y*y)-EY*EY = 0.248
> A=0
> for(i in 1:2)for(j in 1:2)A <- A+p[i,j]*x[i]*y[j]
> EXY<-A
> EXY = 2.16
> COVXY = EXY-EX*EY = -0.104
```
Hence: $\mu_X = \boxed{1.47}$, $\mu_Y = \boxed{1.54}$, $\sigma_X^2 = \boxed{0.249}$, $\sigma_Y^2 = \boxed{0.248}$, $\sigma_{XY} = \boxed{-0.104}$

X and Y are negatively correlated.

(d) $\mu_Z = E[Z] = 1 - E[X] + 2E[Y] = 1 - 1.47 + 2 * 1.54 = \boxed{2.61}$

$var(Z) = var(X) + 4\,var(Y) - 4\,cov(X,Y) = 0.249 + 4 * 0.248 + 4 * 0.104 = \boxed{1.657}$

(3) [6 Pts] Let X be a normal random variable with mean $\mu = 9$ and standard deviation $\sigma = 4$.

(i) Calculate the probability $P(X > 11.5)$

(ii) Calculate the probability $P(|X - 9| \leq 5)$

(iii) Find the value x_c such that $P(X > x_c) = 0.8810$

$X \sim N(\mu = 9, \sigma = 4)$

Using R:

(i) $P(X > 11.5) = 1 - P(X \leq 11.5) = 1 - \texttt{pnorm}(11.5, \texttt{mean} = 9, \texttt{sd} = 4) = \boxed{0.266}$

(ii) $P(|X - 9| \leq 5) = P(4 \leq X \leq 14)$

$= \texttt{pnorm}(14, \texttt{mean} = 9, \texttt{sd} = 4) - \texttt{pnorm}(4, \texttt{mean} = 9, \texttt{sd} = 4) = \boxed{0.789}$

(iii) $P(X > x_c) = 1 - P(X \leq x_c) = 0.8810$

$\Rightarrow P(X \leq x_c) = 1 - 0.8810 = 0.1190$

$\Rightarrow x_c = \texttt{qnorm}(0.119, \texttt{mean} = 9, \texttt{sd} = 4) = \boxed{4.280}$

(4) [4 Pts] Let $\overline{X}$ be the mean of a random sample of size $n = 75$ from an uniform distribution on the interval $-2 \leq x \leq 6$. Use the central limit theorem to approximate the probability $P(2.2 < \overline{X} < 2.6)$.

$$E[X] = \frac{a+b}{2} = 2 = E[\overline{X}]$$

$$var[X] = \frac{(b-a)^2}{12} = \frac{16}{3} \qquad \Rightarrow \sigma^2_{\overline{X}} = \frac{var[X]}{n} = \frac{16}{225} \qquad \Rightarrow \sigma_{\overline{X}} = \frac{4}{15}$$

$P(2.2 < \overline{X} < 2.6) = \texttt{pnorm}(2.6, \texttt{mean} = 2, \texttt{sd} = 4/15) - \texttt{pnorm}(2.2, \texttt{mean} = 2, \texttt{sd} = 4/15) = \boxed{0.214}$

(5) [6 Pts] let X be a continuous random variable with pdf $f(x) = 6x(1-x)$ for $0 \leq x \leq 1$.

(i) Compute the mean and variance of X.

(ii) Let $\overline{X}$ be the mean of a random sample of size $n = 50$ from the pdf given above. Compute the mean and variance of $\overline{X}$.

(iii) Compute the probability $P(0.45 < \overline{X} < 0.55)$.

(i) By direct calculation

$$E[X] = \int_0^1 6x^2(1-x)dx = \int_0^1 (6x^2 - 6x^3)dx = \frac{6}{3} - \frac{6}{4} = \boxed{\frac{1}{2} = 0.5}$$

$$var[X] = \int_0^1 6x^3(1-x)dx - \frac{1}{4} = \int_0^1 (6x^3 - 6x^4)dx - \frac{1}{4} = \frac{6}{4} - \frac{6}{5} - \frac{1}{4} = \boxed{\frac{1}{20} = 0.05}$$

(ii) From (i), it follows that

$$\mu_{\overline{X}} = E[X] = \boxed{\frac{1}{2}}, \qquad \sigma^2_{\overline{X}} = \frac{var[X]}{n} = \boxed{0.001}$$

(iii) $P(0.4 < \overline{X} < 0.6) =$

$\texttt{pnorm}(0.55, \texttt{mean} = 1/2, \texttt{sd} = \texttt{sqrt}(0.001)) - \texttt{pnorm}(0.45, \texttt{mean} = 1/2, \texttt{sd} = \texttt{sqrt}(0.001)) = \boxed{0.886}$