Math 3339 Name: SOLUTION

Test #3

You must justify your work to receive credit. **Whenever you use R, you must report the command you use with the complete set of parameters. You must also report the numerical values of the R output that you use to solve the problem.**

(1)[8 Pts] A study reports that the average tensile strength of a rubber compound is $\mu_0 = 50$ kg/cm^2. A sample of size $n = 12$ was collected, yielding the following values

$$49.7, 49.6, 50.2, 48.1, 50.3, 52.2, 51.8, 52.6, 51.7, 50.5, 51.3, 51.5 \quad (kg/cm^2)$$

(a) Compute a 99% confidence interval of the mean of the tensile strength of the rubber compound. Round your solution to 2 decimal digits.

(b) Under the assumption that the population is normal, test the hypothesis that the average tensile strength of the compound is more than $\mu_0 = 50$ using $\alpha = 0.01$. **NOTE: You must state the hypothesis testing problem and what conclusion you draw from the test**. Round your solution to 3 decimal digits.

(c) What is the minimal value of the significance level α at which we are able to reject the null hypothesis of the hypothesis testing problem in part (b)?

SOLUTION PROBLEM 1

(a) Since data are normal and the variance is unknown, the sample mean will be modeled using the t distribution. We can use `t.test` to find the confidence interval

`t.test(x,mu=50,alternative = "two.sided",conf.level = 0.99)`

From the output we find:

99 percent confidence interval: [49.63, 51.95]

Alternate solution (b):

Data: $\bar{x} = \texttt{mean(x)} = 50.79$, $s = \texttt{sqrt(var(x))} = 1.293$, $t_{0.005,11} = \texttt{qt}(0.995, 11) = 3.106$

Confidence interval:

$\bar{x} \pm t_{0.01/2,11} \frac{s}{\sqrt{12}} = 50.79 \pm \frac{(3.106)(1.293)}{\sqrt{12}} = [49.63, 51.95]$

(b) Since data are normal and the variance is unknown, we will run a t test to solve the hypothesis testing problem.

We test the hypothesis

$H_0 : \mu = 50$;

$H_1 : \mu > 50$.

We can use `t.test` to solve this right-tailed problem

`t.test(x,mu=50,alternative = "greater")`

From the output we find that: p-value = 0.02873 $\simeq$ 0.029.

Since p-value is larger than $\alpha = 0.01$, **we do not reject** H_0.

Alternate solution (c):

Data: $\bar{x} = \texttt{mean(x)} = 50.79$, $s = \texttt{sqrt(var(x))} = 1.293$, $t_{0.01,11} = \texttt{qt}(0.99, 11) = 2.718$

Test statistic $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{50.79-50}{1.293/\sqrt{12}} = 2.117$

Since $t < t_{0.01,11}$, then H_0 cannot be rejected.

p-value: `1- pt(2.117,11)` = 0.0289

(c) The **p-value** = 0.02873 $\simeq$ 0.029 **is the minimal value** of α at which we are able to reject the null hypothesis in this hypothesis testing problem.

(2)[8 Pts] A farmer is studying a new variety of oat seed that is expected to withstand drought better than other varieties.

(a) To estimate the germination of the new oat seed, 300 such seeds are tested and 221 were found to have germinated. Compute a 95% confidence interval for the true proportion of seeds that have germinated. Round your solution to 3 decimal digits.

(b) Compute the sample size n needed to ensure that we are 95% confident to be within 5% (± 0.05) units of the true proportion p.

(c) The farmer knows the oat germination for the parent plants is 78%, but does not know the oat germination for the new hybrid. Use a 5% level of significance to test the hypothesis that the germination rate of the new variety of oat seed is less than 78%. **NOTE: You must state the hypothesis testing problem you are solving and what conclusion you draw from your test.**

SOLUTION PROBLEM 2

(a) $\hat{p} = \frac{221}{300} = 0.737$

For $\alpha = 0.05$, $z_{\alpha/2} = \texttt{qnorm(0.975)} = 1.960$

95% confidence interval: $\hat{p} \pm z_{\alpha/2}\frac{1}{2\sqrt{n}} = 0.737 \pm \frac{1.960}{2\sqrt{300}} = \mathbf{[0.680, 0.783]}$

(b) $n \geq (\frac{z_{\alpha/2}}{2h})^2 = (\frac{1.960}{2(0.05)})^2 = 384.16$. Hence, we can choose $\mathbf{n = 385}$.

(c) We test the hypothesis

$H_0 : p \geq 0.78;$

$H_1 : p < 0.78.$

We apply the R command `prop.test`:

`prop.test(221,300,p=0.78,alternative = "less")`

We find: p-value = 0.04074 (p-value = 0.035 if you used `correct = FALSE`)

Thus, **we accept** H_1 at significance level $\alpha = 0.05$

Alternate solution (c):

Data: $n = 300$, $x = 221$, $\hat{p} = \frac{221}{300} = 0.737$.

Test statistic (Standard Normal pdf):

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.737 - 0.78}{\sqrt{\frac{(0.78)(0.22)}{300}}} = -1.798$$

Rejection region: $z < -z_{0.05} = -qnorm(0.95) = -1.645$

Since $z < -z_{0.05}$, then **we accept** H_1 and H_0 **is rejected**.

(3)[5 Pts] Drunk driving is one of the main causes of car accidents, especially because alcohol intake affects the reaction time of a driver. A sample of 42 drivers was chosen, and their reaction times in an obstacle course were measured after drinking two beers, resulting in the sample mean $\bar{x} = 5.52$ (seconds) with sample variance $s^2 = 1.23$.

(a) Test the hypothesis that the average reaction time is greater than 5 sec using significance level 0.01. **NOTE: You must state the hypothesis testing problem you are solving and what conclusion you draw from your test.**

(b) Compute the p-value of the test.

——————— SOLUTION PROBLEM 3 ———————

(a) We want to compare the average reaction time versus $\mu = 5$.
We test the hypothesis
$H_0 : \mu \leq 5;$
$H_1 : \mu > 5.$

Test statistic:

$$Z = \frac{5.52 - 5}{\sqrt{1.23/42}} = 3.038613$$

Since $n > 30$ and the data distribution is not known, we use a z-test. For $alpha = 0.01$,

$$z_{0.01} = \texttt{qnorm}(1 - 0.01) = 2.326348$$

Since $3.038613 > z_{0.01}$, **we reject H_0 at significance level 0.01.**

(b) Since this is a one-sided upper tailed test, the p-value is computed as follows

p-value $= P(z \geq Z) = 1 - P(z \leq Z) = 1 - \texttt{pnorm}(3.038613) = \mathbf{0.00118835}$