Test #1

Please, write clearly and justify your work to receive credit. If you use R, you need to report the R command you entered with the complete list of parameters. You also need to report the R output that you used to draw your conclusions.

(1) [5Pts] The following data are the oxygen uptakes (milliliters) during incubation of a random sample of 13 cell suspensions:

(a) Use an appropriate statistical test to check whether data can be assumed to satisfy the normality assumption.
(b) Do these data provide sufficient evidence at the 0.01 level of significance that the population mean is not 12 ml? When you solve the problem, you must state the hypothesis testing problem you are solving and justify your conclusion.

(a)
> shapiro.test(x)
Shapiro-Wilk normality test
data: x
W = 0.96688, p-value = 0.8544
Conclusion: Since the p-value > 0.05, we can assume that data are normally distributed.

(b) We test $H_0 : \bar{x} = 12$ against $H_1 : \bar{x} \neq 12$ with $\alpha = 0.01$.
From the data: $\bar{x} = 13.331$, $s^2 = 1.842$, $s = 1.357$, $n = 13$.
Test statistic (Student t pdf):
$$t = \frac{\bar{x}_r - x_0}{\sqrt{\frac{s^2}{n}}} = \frac{13.331 - 12.000}{\sqrt{\frac{1.842}{13}}} = 3.535$$
Rejection region: $t > t_{0.01/2,12} => qt(1 - 0.01/2, 12) = 3.055$
Conclusion: since $t > t_{0.01/2,12}$, then $H_0$ is REJECTED.

R solution
> t.test(x,mu=12,alternative="two.sided")
One Sample t-test
data: x
t = 3.535, df = 12, p-value = 0.004108
alternative hypothesis: true mean is not equal to 12
Conclusion: Since p-value is less that 0.01, then $H_0$ is REJECTED.

(2) [4Pts] A study about the effects of reminiscence therapy for older women with depression considers a sample of 10 women residing in an assisted living long-term care facility. For this study, depression was measured by the Geriatric Depression Scale (GDS). Higher scores indicate more severe depression symptoms. The participants received reminiscence therapy. Pre-treatment and posttreatment depression scores are given in the following table.

Pre–GDS: 12, 10, 16, 2, 12, 18, 11, 16, 16, 10
Post–GDS: 11, 10, 11, 3, 9, 13, 8, 14, 14, 10
Can we conclude that subjects who participate in reminiscence therapy experience, on average, a decline in GDS depression scores? Let $\alpha = 0.01$. When you solve the problem, you must state the hypothesis testing problem you are solving and justify your conclusion.

We apply a Paired t-test. We set $d = x_{\text{pre}} - x_{\text{post}}$. We test $H_0 : \mu_d \leq 0$ against $H_1 : \mu_d > 0$ with $\alpha = 0.01$.

Data: $n = 10, \bar{d} = \frac{1}{10} \sum_{i=1}^{15} d_i = 2, s_d = 2.055$.

Test statistic (Student t pdf):

$$t = \bar{d} - \mu_d \frac{s_d}{\sqrt{n}} = \frac{2}{2.055/\sqrt{10}} = 3.078$$

Rejection region: $t > t_{0.005,9} = 2.821$
Since $t > t_{0.005,9}$, then $H_0$ is REJECTED.

R solution
> x=c(12, 10, 16, 2, 12, 18, 11, 16, 16, 10)
> y=c(11, 10, 11, 3, 9, 13, 8, 14, 14, 10)
> t.test(x,y,alternative = "greater", paired = TRUE,var.equal = TRUE)

Paired t-test
data: x and y
t = 3.0779, df = 9, p-value = 0.006592
alternative hypothesis: true mean difference is greater than 0

Conclusion: Since p-value is less that 0.01, then $H_0$ is REJECTED.

(3) [7Pts] A study of Marfan syndrome (a genetic disorder that affects connective tissue) reported the following severity scores of patients with no dural ectasia (NO), mild dural ectasia (MI) and marked dural ectasia (MA):

- **NO**: 18, 18, 20, 21, 23, 23, 24, 26, 26, 27, 28, 20, 29, 21, 20, 21, 20, 18, 10, 16, 22, 22, 23, 26, 28, 28, 28, 29, 29, 30, 31, 32, 32, 33, 33, 38, 39, 40, 47
- **MI**: 10, 16, 22, 22, 23, 26, 28, 28, 29, 29, 30, 31, 32, 32, 33, 33, 38, 39, 40, 47
- **MA**: 17, 24, 26, 27, 29, 30, 30, 33, 34, 35, 35, 36, 39

(a) May we conclude, on the basis of these data, that mean severity scores differ among the three populations represented in the study? Let $\alpha = 0.05$.
(b) Use Tukey's procedure to test for significant differences among individual pairs of sample means.
(c) Verify that normality of the data and homogeneity of the variance are satisfied using appropriate statistical tests.

> severity <-c(18, 18, 20, 21, 23, 23, 24, 26, 26, 27, 28, 20, 29, 21, 20, 21, 20, 18, 10, 16, 22, 22, 23, 26, 28, 28, 28, 29, 29, 30, 31, 32, 32, 33, 33, 38, 39, 40, 47, 17, 24, 26, 27, 29, 30, 30, 33, 34, 35, 35, 36, 39)
> group <-factor(c(rep("NO",len=18),rep("MI",len=21),rep("MA",len=13)))
> Data <- data.frame(severity, group)
> str(Data)
'data.frame': 52 obs. of 2 variables:
$ severity: num 18 18 20 21 23 23 24 26 26 27 ...
$ group : Factor w/ 3 levels "MA","MI","NO": 3 3 3 3 3 3 3 3 3 3 ...
Alternatively, if data are loaded from the file test1.csv

```r
> Data <- read.csv("C:/Users/dlabate/Desktop/Teaching/ma4310/test1.csv")
> Data$group <- factor(Data$group, levels = c(1,2,3),labels = c("NO", "MI","MA"))
> res.aov <- aov(severity ~ group, data = Data)
> str(test1)
'data.frame': 52 obs. of 2 variables:
$ severity: int 18 18 20 21 23 23 24 26 26 27 ...
$ group : Factor w/ 3 levels "NO","MI","MA": 1 1 1 1 1 1 1 1 1 1 ...

(a)
> res.aov <- aov(severity ~ group, data = Data)
> summary(res.aov)

            Df Sum Sq Mean Sq  F value Pr(> F) 
group       2 644.1 322.00    7.937  0.0010 **
Residuals   49 1988.0 40.68

Conclusion: Since p-value is less that 0.05, then \( H_0 \) is REJECTED, that is, mean severity scores differ among the three populations.

(b)
> TukeyHSD(res.aov)
Tukey multiple comparisons of means
95% family-wise confidence level

Fit: aov(formula = severity ~ group, data = Data)

$group  diff lwr  upr  padj
MI - MA -1.051282 -6.484201 4.381637  0.886711
NO - MA -7.995726 -13.599080 -2.392373  0.003288
NO - MI -6.944444 -11.889390 -2.004499  0.003854

Conclusion: only the differences NO-MA and NO-MI are significant but the difference MI-MA is not.

(c)
We test normality first:
> aov.residuals <- residuals(object = res.aov)
> shapiro.test(x = aov.residuals)

Shapiro-Wilk normality test
data:  aov.residuals
W = 0.96338, p-value = 0.1097
Since the p-value is above 0.05, we can assume normality of the data.
Finally we test homogeneity.
> library(car)
> leveneTest(severity ~ group, data = Data)

Levene’s Test for Homogeneity of Variance (center = median)
<table>
<thead>
<tr>
<th></th>
<th>df</th>
<th>F value</th>
<th>Pr(&gt; F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td>2</td>
<td>2.4294</td>
<td>0.09863</td>
</tr>
</tbody>
</table>

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Since the p-value is above 0.05, can assume the homogeneity of variances.