HW #1

Let \( v_1 = (v_1, v_2) \) and \( u_1 = (u_1, u_2) \) be vectors in \( \mathbb{C}^2 \) and let \( M = \begin{pmatrix} 2 & -i \\ i & 3 \end{pmatrix} \). Prove that

\[
\langle u, v \rangle = (\overline{u_1}, \overline{u_2}) M \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}
\]

defines an inner product on \( \mathbb{C}^2 \).

(2) Show that the inner product of \( L^2([a, b]) \) defined by

\[
\langle f, g \rangle = \int_a^b f(t) \overline{g(t)} \, dt,
\]
is conjugate-symmetric, homogeneous and linear.

This problem is part of Exercise 4 at p. 35 in the textbook. You are encouraged to work out the whole problem on your own, but I will collect the part indicated above. The solution of the positivity part is given in the back of the book.