MATH 4377/6308 - Advanced linear algebra I - Summer 2024
Quiz 1

(1) [6 Pts] Let \( f : [0, 2\pi] \to [-1, 1] \) be defined by \( f(x) = \cos(x) \).

a) Is \( f \) one-to-one? Is \( f \) onto?

b) Find an interval \( S \), such that \( f|_S \) is both one-to-one and onto. Sketch the function on \( S \).

SOLUTION:
(a) \( f \) is not 1-1 since \( f(0) = f(\pi) \). \( f \) is onto.
(b) \( f \) one-to-one and onto in the interval \([0, \pi]\) where it is monotonically decreasing.

(2) [4 Pts] Let \( x, y \in \mathbb{Z} \). Let \( x \sim y \) if and only if \( y + 4x \) is an integer multiple of 5. Prove that \( \sim \) is a transitive relation.

SOLUTION:
If \( y + 4x = 5m \) and if \( z + 4y = 5n \), then (using these two equations to express \( z \) and \( 4x \)) \( z + 4x = (5n - 4y) + (5m - y) = 5(n + m) - 5y = 5(n + m - y) \), which is a multiple of 5.