Please send Enrichmenats as pdfs to me for comments. They are actually completion graded and are used to provide guidance and feedback.

Summer grader email: TBA

Summer grader website: TBA

I will be grading them! The grader will be in charge of posting them to his website.

**Introduction:**

Create a snowman that says something about who you are. Instructions are on my website. This is a good introduction to Sketchpad – and a reminder that winter is really going to come, just later after the course ends!

**Chapter 2**

**Enrichment 1** Find two more models for A Flexible Geometry.

Sketch by hand or in Sketchpad.

**A Flexible Geometry not from the book**

**Undefined terms:** point, line, on

**Axioms:** A1 Every point is on exactly two distinct lines.

A2 Every line is on exactly three distinct points.

Models:

There are **lots** of very different models for this geometry.

Here are two:

One has a finite number of points and the other has an infinite number of points,

so they are non-isomorphs.

**Model 1**: 3 points, 2 lines



Points are dots and lines are S-curves. One line

is dotted so you can tell it from the second line.

Nobody ever said “lines” have to be straight things, you know.

Note, too, that there are only 3 points so my lines are composed of some material that is NOT points, it’s “line stuff”. Some non-point stuff.

Luckily they’re undefined terms so I don’t have to go into it.

**Model 2**: an infinite number of points and lines

This is an infinite lattice. Each line is has 3 points along it. It continues forever left and right

**Enrichment 2**

Q1 Prove the first 2 theorems in Fano’s Geometry.

Q2 What is the situation with respect to parallel lines in Fano’s Geometry?

**The Seven-point geometry**

Also known as Fano’s geometry. (Gino Fano, published 1892)

Axioms for Fano's Geometry

Undefined Terms. *point, line,* and *incident*.

Axiom 1. There exists at least one line.

Axiom 2. Every line has exactly three points incident to it.

Axiom 3. Not all points are incident to the same line.

Axiom 4. There is exactly one line incident with any two distinct points.

Axiom 5. There is at least one point incident with any two distinct lines.

Model on next page

{BDF} is a line! Nobody said “straight” in the axioms!

Where does {BDF} intersect {CBA}?

Theorem 1. Two distinct lines intersect in exactly one point.

Theorem 2. Fano's geometry consists of exactly seven points.

Theorem 3.Each point lies on exactly three lines.

Theorem 4. The lines through any one point of the geometry contain all the points of the geometry.

Theorem 5. For any pair of points in the geometry, there exist exactly two lines not containing either point.

Theorem 6. For a set of three lines not all containing the same point, there exists exactly one point in the geometry not on any of the three lines.

**Enrichment 3**

Come up with one model and prove one of the three theorems. If you pick a theorem other than T1 you may use the preceding Theorem(s) in your proof.

Howard Eves’ Geometry

Undefined terms: line points

Axioms: A1 Every line is a collection of at least two points.

A2 There exist at least two points.

A3 If A and B are two distinct points, then there exists one and only one line containing both of them.

A4 If L is a line, then there exists a point A not in L.

A5 If L is a line and A is a point not in L, then there exists one and only one line containing A and not containing any point that is in L.

Theorems:

T1 Every point is contained in at least two lines.

T2 There exist at least 4 distinct points.

T3 There exist at least six distinct lines.

**Chapter 3**

**Enrichment 4 Taxicab Geometry**

In Euclidean Geometry, circles have 3 possible relationships:

* they don’t intersect at all
* they intersect in one point

they can be internally tangent or externally tangent

* they intersect in two points

What are the analogous situations for TCG circles? In what way can two TCG circles interact that is impossible for EG circles?

**Enrichment 5 Spherical Geometry**

Google SMSG axioms for Euclidean Geometry. Compare and contrast them to the axioms in our notes on pages 1 and 2 in the 3.3 ½ notes. What do the differences give you or point out to you?

One page, front side only, typed, double spaced.

**Enrichment 6**

Find an internet site that discusses Saccheri Quadrilaterals in a way that appeals to you. Summarize the site, noting what was interesting and give the URL as the reference.

Chapter 4

**Enrichment 7**

See the text page 240 (1) – (5) in the Pythagorean Theorem derivation.

Write out the proof in agonizing detail. Show every little detail and step for how and why this works.

**Enrichment 8**

Prove the 3 bulleted Corollaries to the Inscribed Angle Theorem on pages 272 and 273: They are illustrated in Figure 4.61

Enrichment 9:

Do the Moment for Discovery in Absolute Geometry on page 433

Be sure to write it up nicely with illustrations and complete answers. Feel free to check with any other student in the class to make sure you’ve “got it”.