Mathematics 3336: Review #1

Chapters 1 and 2 have review problems at the end. Focus on the types of questions that are similar to those given in homework assignments. Those of particular importance are

Chapter 1: 3, 6, 7, 8, 10, 16  
Chapter 2: 1, 2, 4, 6, 9, 10, 16

1. What is a proposition?

Expression with truth value

2. Give an example of an expression that isn’t a proposition.

“This sentence is false.”

3. Explain the conjunction, and disjunction of two propositions.

\( \wedge \) : conjunction, \ AND \  \( \vee \) : disjunction, \ OR

4. What is a predicate?

Evaluates variable as true or false.

5. What are the two standard quantifiers? Give their names and symbols.

\( \forall \) : universal \  \( \exists \) : existential

6. How do quantifiers make expressions propositions?

It closes the expression so it has truth value

7. How are sets defined?

Entirely by what elements it has, \( \exists a, b, c \exists \)

set builder, \( \exists x : P(x) \exists \)

8. What is the notation for an object being an element of a set? A set being a subset of a set? A set being a proper subset of a set?

\( x \in A, A \subseteq B, A \subset B \)

9. What are the common operations on sets? Please express in set builder notation and logic.

\( A \cup B, A \cap B, A \setminus B \)

\( \exists x : x \in A \times x \in B \), \( \exists x : x \in A \land x \in B \), \( \exists x : x \in A \land x \notin B \)

10. What is the power set of a given set?

\( P(A) = \exists B : B \subseteq A \exists \)

11. What is Russell’s Paradox?

\( \exists x : x \notin x \exists \)

12. What is the most commonly used set of axioms for an axiomatic set theory? How does it approach resolving Russell’s Paradox?

\( ZF \) (Zermelo-Fraenkel)

Axiom of regularity/foundation + Axiom of restricted comprehension (Specification)
13. What is the definition of a function? Use set theory for your explanation.

\[ f \subseteq X \times Y \text{ s.t. every pair } (x, y) \text{ with } (a, b) = (x, y) \forall a \in X \]

\[ \iff f : X \rightarrow Y, f(x) = y \iff (x, y) \in f \]

14. What does it mean for a function to be injective, surjective, or bijective?

1 to 1, onto, or both

\[ f(x) = f(y) \Rightarrow x = y, \forall y \in Y \exists x \in X [f(x) = y] \]

15. Given two sets, how can one tell if they are the same cardinality?

\[ |X| = |Y| \iff \exists \text{ a bijection } f : X \rightarrow Y \]

16. Given two sets not of the same cardinality, how do you determine which has the larger cardinality?

\[ |X| \geq |Y| \iff \exists \text{ a surjection } p : X \rightarrow Y \]

17. What is the definition of a set being finite or infinite?

A set \( X \) is finite if and only if \( \exists S \subseteq X \) with \( |S| = |X| \)

18. What does it mean to say that a set is countably infinite?

\[ |X| = |\mathbb{N}| \]

It can be enumerated.