1. What is a permutation? What is a combination? What are their formulations?

\[ P(n, k) = \frac{n!}{(n-k)!} \quad \text{number of ways of arranging } k \text{ things out of } n. \]

\[ C(n, k) = \frac{n!}{k!(n-k)!} \quad \text{number of ways of choosing } k \text{ objects out of } n. \]

2. What is Pascal’s Identity? Pascal’s Triangle?

\[ \binom{n}{k} = \binom{n-1}{k-1} + \binom{n}{k-1} \]

3. What is the Binomial Theorem?

\[ (a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i \]

4. What is a measure? What are the two rules a measure must have?

Assign a size to a set. \( \mu: S \rightarrow \mathbb{R} \)

- \( \mu(\emptyset) = 0 \)
- \( \mu\left( \bigcup_{i=1}^{\infty} A_i \right) = \sum_{i=1}^{\infty} \mu(A_i) \) when \( A_i \) disjoint.

5. Give three or more different examples of a measure.

Counting measure, Probability, Length, Area, Volume, ...

6. What condition makes a measure a probability measure?

\[ P(\Omega) = 1 \quad \text{probability measure of the full space is 1.} \]

7. What is the inclusion-exclusion principle for measurable sets?

\[ \mu\left( \bigcup_{j=1}^{N} A_j \right) = \sum_{j=1}^{N} \mu(A_j) - \sum_{1 \leq i_1 < \cdots < i_k \leq N} \mu(A_{i_1} \cap \cdots \cap A_{i_k}) + \sum_{1 \leq j_1 < \cdots < j_l \leq N} \mu(A_{j_1} \cap \cdots \cap A_{j_l}) - \cdots \pm \mu(A_1 \cap \cdots \cap A_N) \]

8. What is a relation?

\[ R \subseteq A \times B \quad \text{a set of ordered pairs } (a,b) \in A \times B \]

9. What does it mean for a relation to be

1. Reflexive

\[ \forall a \in A \quad (a,a) \in R \]

2. Symmetric

\[ (a,b) \in R \Rightarrow (b,a) \in R \]

3. Antisymmetric

\[ (a,b) \in R \land (b,a) \in R \Rightarrow a = b \]

4. Transitive

\[ (a,b) \in R \land (b,c) \in R \Rightarrow (a,c) \in R \]

5. Left-Total

\[ \forall a \in A \exists (a,b) \in R \]

6. Right-Total

\[ \forall b \in B \exists (a,b) \in R \]

7. Functional

\[ (a,b) \in R \land (a,c) \in R \Rightarrow b = c \]

8. Injective

\[ (a,b) \in R \land (c,b) \in R \Rightarrow a = c \]
10. What properties do the following relations have?
   1. Function
      - Left Total & Functional
   2. Multi-function
      - Left Total
   3. Equivalence Relation
      - Reflexive, Symmetric, Transitive
   4. Partial Order
      - Reflexive, Antisymmetric, Transitive

11. What is the matrix representation of a relation?
    \[ M_R \ni m_{ij} = \begin{cases} 1 & (a_i, b_j) \in R \\ 0 & \text{otherwise} \end{cases} \]

12. What is an undirected graph? Similarly what is a directed graph?
   - Undirected graph: \( G = (V, E) \), edges connect vertices.
   - Directed graph: \( G = (V, E) \), edges connect a start to an end.

13. What is the adjacency matrix of a graph?
    \[ A \ni a_{ij} = \# \text{ of edges connecting } v_i \text{ to } v_j \]