1. The solution set of the system
\[\begin{align*}
  x + 2y - 3z + 4w &= 2 \\
  2x + 5y - 2z + w &= 1 \\
  5x + 12y - 7z + 7w &= 4
\end{align*}\]
is:
(a) no solutions
(b) \(x = -8 + 11a, \ y = 2 - 4a, \ z = a, \ w = a, \ a \) any real number
(c) \(x = 8 + 11a, \ y = -3 - 4a, \ z = a, \ w = 0, \ a \) any real number
(d) \(x = 8 + a, \ y = -3 + 4a, \ z = 1, \ w = a, \ a \) any real number
(e) None of the above.

2. The solution set of the system
\[\begin{align*}
  2x + 8y + 11z &= 7 \\
  x + 4y + 3z &= 1 \\
  x + 6y + 7z &= 3
\end{align*}\]
is:
(a) \(x = 1, \ y = -1, \ z = 2\)
(b) no solution
(c) \(x = 2 - a, \ y = -3 - 2a, \ z = a, \ a \) any real number
(d) \(x = 2, \ y = -1, \ z = 1\)
(e) None of the above.

3. The solution set of the system
\[\begin{align*}
  3x + 6y - 3z &= 6 \\
  -2x - 4y - 3z &= -1 \\
  3x + 6y - 2z &= 10
\end{align*}\]
is:
(a) \(x = 7 - a, \ y = 2 + a, \ z = a, \ a \) any real number.
(b) \(x = 3 - 2a, \ y = 4a - 1, \ z = a, \ a \) any real number.
(c) no solution
(d) \(x = 4, \ y = -2, \ z = -1\).
(e) None of the above.
4. The solution set of the system
\[
\begin{align*}
x + 2y - z &= 3 \\
2x + 4y - 2z &= 6 \\
3x + 6y + 2z &= -1
\end{align*}
\]
is:

(a) \( x = 1 - 2a, \ y = a, \ z = -2, \ a \) any real number.
(b) \( x = 3 - 2a, \ y = 4a - 1, \ z = a, \ a \) any real number.
(c) \( x = 2 - 2a, \ y = a, \ z = -1, \ a \) any real number.
(d) \( x = -1, \ y = -2, \ z = -2. \)
(e) None of the above.

5. The solution set of the system
\[
\begin{align*}
2y - 3z + w &= 0 \\
-x + y - z + 4w &= 0 \\
-2x - 2y + 2z - 8w &= 0
\end{align*}
\]
is:

(a) no solutions
(b) \( x = -\frac{1}{2}a - \frac{7}{2}b, \ y = \frac{3}{2}a - \frac{1}{2}b, \ z = a, \ w = b, \ a, b \) any real numbers
(c) \( x = \frac{1}{2}a + \frac{7}{2}b, \ y = -\frac{3}{2}a + \frac{1}{2}b, \ z = a, \ w = b, \ a, b \) any real numbers
(d) \( x = -\frac{1}{2}a - \frac{7}{2}b, \ y = \frac{3}{2}a - \frac{1}{2}b, \ z = a, \ w = 0, \ a \) any real number
(e) None of the above.

6. The solution set of the system
\[
\begin{align*}
x + y - 2z + 3w &= 4 \\
2x + 3y + 3z - w &= 3 \\
5x + 7y + 4z + w &= 5
\end{align*}
\]
is:

(a) no solutions
(b) \( x = 9 + 9a - 10b, \ y = -5 - 7a + 7w, \ z = a, \ w = b, \ a, b \) any real numbers
(c) \( x = 8 + 11a, \ y = -3 - 4a, \ z = a, \ w = 0, \ a \) any real number
(d) \( x = 9 + 9a, \ y = -5 - 7a, \ z = a, \ w = 0, \ a \) any real numbers
(e) None of the above.
Use the system of equations
\[
\begin{align*}
    x - 2y &= 1 \\
    x - y + kz &= -1 \\
    ky + 9z &= 6
\end{align*}
\]
for problems 7 and 8.

7. The value(s) of \( k \) such that the system has a unique solution is (are):
   
   (a) \( k \neq 3 \)
   
   (b) \( k \neq \pm 3 \)
   
   (c) \( k \neq -3 \)
   
   (d) \( k = 3, -3 \)
   
   (e) None of the above.

8. The value(s) of \( k \) such that the system has a infinitely many solutions is (are):
   
   (a) \( k = -3 \)
   
   (b) \( k = 3 \)
   
   (c) \( k = -2 \)
   
   (d) \( k \neq -3 \)
   
   (e) None of the above.

Use the system of equations
\[
\begin{align*}
    x - 4y - 2z &= 2 \\
    y + 2z &= 2 \\
    -2x + 7y + (k^2 - 2)z &= k - 4
\end{align*}
\]
for problems 9 and 10.

9. The value(s) of \( k \) such that the system has a unique solution is (are):
   
   (a) \( k \neq 2 \)
   
   (b) \( k = -1, 2 \)
   
   (c) \( k \neq \pm 2 \)
   
   (d) \( k \neq -2 \)
   
   (e) None of the above.

10. The value(s) of \( k \) such that the system has no solution is (are):
    
    (a) \( k = 2, -3 \)
    
    (b) \( k = 0 \)
    
    (c) \( k = 2 \)
    
    (d) \( k = -2 \)
    
    (e) None of the above.
11. The reduced row echelon form of \[
\begin{pmatrix}
1 & 2 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\] is (a) \[
\begin{pmatrix}
1 & 2 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\] (b) \[
\begin{pmatrix}
1 & 2 & -1 & 0 & 5 \\
0 & 0 & 1 & 0 & 9 \\
0 & 0 & 0 & 1 & 2
\end{pmatrix}
\] (c) \[
\begin{pmatrix}
1 & 2 & -1 & 0 & -4 \\
0 & 0 & 0 & 0 & -2 \\
0 & 0 & 0 & 1 & -1
\end{pmatrix}
\] (d) \[
\begin{pmatrix}
1 & 2 & -1 & 0 & 8 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\] (e) None of the above.

12. The reduced row echelon form of \[
\begin{pmatrix}
0 & 0 & 2 & -2 & 2 \\
3 & 3 & -3 & 9 & 12 \\
4 & 4 & -2 & 11 & 12
\end{pmatrix}
\] is (a) \[
\begin{pmatrix}
1 & 1 & 0 & 0 & 7 \\
0 & 1 & 1 & 0 & -5 \\
0 & 0 & 0 & 1 & -2
\end{pmatrix}
\] (b) \[
\begin{pmatrix}
1 & 1 & -5 & 0 & 22 \\
0 & 1 & 1 & 0 & -5 \\
0 & 0 & 0 & 1 & -2
\end{pmatrix}
\] (c) \[
\begin{pmatrix}
1 & 1 & 0 & 0 & 17 \\
0 & 0 & 1 & 0 & -5 \\
0 & 0 & 0 & 1 & -6
\end{pmatrix}
\] (d) \[
\begin{pmatrix}
1 & 1 & 0 & 0 & 8 \\
0 & 0 & 1 & 0 & -5 \\
0 & 0 & 0 & 1 & -2
\end{pmatrix}
\] (e) None of the above.
13. The ranks of the augmented matrix and the matrix of coefficients of the system of equations

\[
\begin{align*}
x + 2y - 3z &= 1 \\
2x + 5y - 8z &= 4 \\
3x + 8y - 13z &= 7
\end{align*}
\]

are, respectively,

(a) 3, 3
(b) 2, 3
(c) 2, 2
(d) 3, 2
(e) None of the above.

14. The ranks of the augmented matrix and the matrix of coefficients of the system of equations

\[
\begin{align*}
x + 2y - 3z &= -1 \\
3x - y + 2z &= 7 \\
5x + 3y - 4z &= 2
\end{align*}
\]

are, respectively,

(a) 3, 3
(b) 2, 3
(c) 2, 2
(d) 3, 2
(e) None of the above.

15. The system of equation in Problem in 14 has

(a) a unique solution.
(b) infinitely many solutions.
(c) no solution.
(d) uncountably many solutions.
(e) None of the above.