Second Order Nonhomogeneous Differential Equations:  Section 3.4, 3.5

1. \( z_1(x) = 2x^3 + x \ln x, \quad z_2(x) = x \ln x - x^3 \) are solutions of a second order, linear nonhomogeneous equation \( L[y] = f(x) \). \( y_1(x) = x^{-2} \) is a solution of the corresponding reduced equation \( L[y] = 0 \).

   (a) Give a fundamental set of solutions of the reduced equation \( L[y] = 0 \). (Hint: The difference of two solutions of a nonhomogeneous equation is a solution of its reduced equation.)

   (b) Give the general solution of the nonhomogeneous equation \( L[y] = f(x) \).

2. \( z_1(x) = 2x^2 + \tan x, \quad z_2(x) = x^2 - 2x + \tan x, \quad z_3(x) = x^2 - 3x + \tan x \) are solutions of a second order, linear nonhomogeneous equation \( L[y] = f(x) \).

   (a) Give a fundamental set of solutions of the corresponding reduced equation \( L[y] = 0 \). (See the hint in # 1.)

   (b) Give the general solution of the nonhomogeneous equation \( L[y] = f(x) \).

3. Given the differential equation \( y'' + p(x)y' + q(x)y = 4x \). \( \{y_1 = x^2, \quad y_2 = x^2 \ln x \} \) is a fundamental set of solutions of the reduced equation. Find the general solution of the given equation

4. Given the differential equation \( y'' - \frac{5}{x}y' + \frac{8}{x^2}y = 2x^2 \). The reduced equation has solutions of the form \( y = x^r \). Find the general solution of the given equation

5. Find a particular solution of \( y'' - \frac{4}{x}y' + \frac{6}{x^2}y = 6x + 2 \).

6. Find the general solution of \( y'' + 4y = 2 \tan 2x \).

7. Find the general solution of \( y'' - 6y' + 9y = \frac{e^{3x}}{x} \).

8. Find the general solution of \( y'' + 9y = 4 \cos 2x \).

9. Find the general solution of \( y'' + 4y = 2 \sin 2x \).

10. Find the general solution of \( y'' - 6y' + 8y = 2e^{4x} + 6 \).

11. A particular solution of the nonhomogeneous differential equation

\[
y'' - 2y' - 15y = 2 \cos 3x + 5e^{5x} + 2
\]

will have the form:
12. A particular solution of the nonhomogeneous differential equation
\[ y'' - 8y' + 16y = e^{2x} \sin 4x + 2e^{4x} + 5x \]
will have the form:

Higher Order Linear Equations: Section 3.7

1. The general solution of \( y''' - 4y'' + y' + 6y = 0 \) is: (Hint: \( 7e^{2x} \) is a root of the characteristic equation)

2. The general solution of \( y''' + y'' - 8y' - 12y = 0 \) is: (Hint: \( r = 3 \) is a root of the characteristic equation)

3. The general solution of \( y^{(4)} + 2y''' + 4y'' - 2y' - 5y = 0 \) is: (Hint: \( e^{-x} \cos 2x \) is a solution)

4. The homogeneous equation with constant coefficients that has
\[ y = C_1e^{-2x} + C_2xe^{-2x} + C_3 \cos 2x + C_4 \sin 2x + C_5 \]
as its general solution is:

5. The homogeneous equation with constant coefficients of least order that has
\[ y = 2e^{3x} + 3 \sin 2x + 2x \]
as a solution is:

6. A particular solution of \( y''' - 2y'' - 3y' = 2e^{-x} + xe^{3x} + 2 \) will have the form:

7. A particular solution of \( y^{(4)} - 16y = 2e^{-2x} + 3e^{4x} + \cos 2x + 5 \) will have the form:

8. The general solution of \( y^{(4)} + 5y'' - 36y = -2 \cos 3x + 3xe^{2x} \) will have the form:

9. The general solution of \( y''' + y'' + y' + y = 5 \sin x + 2e^x - e^{-x} + 4x \) will have the form:

10. A particular solution of \( y^{(4)} - 5y''' + 7y'' - 3y' = 2e^x - 4xe^{3x} + 7 \) will have the form:

Laplace Transformations: Chapter 4

1. Find the Laplace transform of \( f(x) = 2e^{-3x} + \cos 2x + 5x \).

2. Find the Laplace transform of \( f(x) = 3x e^{2x} + e^x \sin 3x \).

3. If \( F(s) = \frac{2}{s^2} + \frac{s - 3}{s^2 + 4} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

4. If \( F(s) = \frac{4}{s^4 - 3s^3 + 2s^2} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

5. If \( F(s) = \frac{3s^3 + 6s^2 + 36}{s^4 + 9s^2} \), then \( \mathcal{L}^{-1}[F(s)] \) is:
6. If \( F(s) = \frac{2s + 1}{(s + 3)(s^2 + 1)} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

7. If \( F(s) = \frac{2s + 1}{s^3 - s^2 - 8s + 12} \), then \( \mathcal{L}^{-1}[F(s)] \) is: (Hint: 2 is a root)

8. If \( F(s) = \frac{3s + 1}{s^3 - 6s^2 + 13s - 20} \), then \( \mathcal{L}^{-1}[F(s)] \) is: (Hint: 4 is a root)

9. Find the Laplace transform of the solution of the initial-value problem

\[
y' + 2y = 3 \cos 2x; \quad y(0) = 3.
\]

10. Find the Laplace transform of the solution of the initial-value problem

\[
y'' - 5y' + 6y = 4 \sin 3x; \quad y(0) = 0, \quad y'(0) = 2.
\]

11. Find the Laplace transform of the solution of the initial-value problem

\[
y'' + 25y = 2e^{-3x}; \quad y(0) = 2, \quad y'(0) = 0.
\]

12. Use the Laplace transform method to find the solution of the initial-value problem

\[
y' - 3y = 2e^{2x}; \quad y(0) = 1.
\]

13. Use the Laplace transform method to find the solution of the initial-value problem

\[
y' + 4y = 3 \cos 2x; \quad y(0) = 3.
\]

14. Use the Laplace transform method to find the solution of the initial-value problem

\[
y'' - 3y' + 2y = 2x + 1, \quad y(0) = 2, \quad y'(0) = -1.
\]

15. Find the value(s) of \( \gamma \) such that the solution of the initial-value problem

\[
y'' - 4y = \sin x; \quad y(0) = \gamma, \quad y'(0) = 0
\]

is bounded on \([0, \infty)\).

16. Find the value of \( \delta \) such that the solution of the initial-value problem

\[
y' - 3y = 2e^{-2x}; \quad y(0) = \delta
\]

has limit 0 as \( x \to \infty \).

17. If

\[
f(x) = \begin{cases} 
  x^2 + 1 & 0 \leq x < 3 \\
  2x & x \geq 3 
\end{cases}
\]

then \( \mathcal{L}[f(x)] = \)
18. If
\[ f(x) = \begin{cases} \sin x & 0 \leq x < \pi/2 \\ \cos 2x & x \geq \pi/2 \end{cases} \]
then \( \mathcal{L}[f(x)] = \)

19. If
\[ f(x) = \begin{cases} -2 & 0 \leq x < 2 \\ x & 2 \leq x < 5 \\ 3 & x \geq 5 \end{cases} \]
then \( \mathcal{L}[f(x)] = \)

20. If
\[ f(x) = \begin{cases} x^2 + 1 & 0 \leq x < 2 \\ 4e^{3x} & x \geq 2 \end{cases} \]
then \( \mathcal{L}[f(x)] = \)

21. If \( F(s) = \frac{2}{s} + \frac{4}{s^3} - 2e^{-3s} \frac{1}{s^2} + 4e^{-2s} \frac{1}{(s+2)(s^2-2s+10)} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

22. If \( F(s) = \frac{s + 4e^{-3s}}{s^3 - 2s^2} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

23. If \( F(s) = \frac{3s + 1}{s^2 - s - 6} + \frac{(2s - 4)e^{-4s}}{s^2 - 2s + 10} \), then \( \mathcal{L}^{-1}[F(s)] \) is: