Second Order Nonhomogeneous Differential Equations: Section 3.4, 3.5

1. \( z_1(x) = 2x^3 + x \ln x, \quad z_2(x) = x \ln x - x^3 \) are solutions of a second order, linear nonhomogeneous equation \( L[y] = f(x), \quad y_1(x) = x^{-2} \) is a solution of the corresponding reduced equation \( L[y] = 0. \)

   (a) Give a fundamental set of solutions of the reduced equation \( L[y] = 0. \)

   (b) Give the general solution of the nonhomogeneous equation \( L[y] = f(x). \)

2. \( z_1(x) = 2x^2 + \tan x, \quad z_2(x) = x^2 - 2x + \tan x, \quad z_3(x) = x^2 - 3x + \tan x \) are solutions of a second order, linear nonhomogeneous equation \( L[y] = f(x). \)

   (a) Give a fundamental set of solutions of the corresponding reduced equation \( L[y] = 0. \)

   (b) Give the general solution of the nonhomogeneous equation \( L[y] = f(x). \)

3. Given the differential equation \( y'' + p(x)y' + q(x)y = 4x. \) \( \{y_1 = x^2, \ y_2 = x^2 \ln x\} \) is a fundamental set of solutions of the reduced equation. Find the general solution of the given equation.

4. Given the differential equation \( y'' - \frac{5}{x}y' + \frac{8}{x^2}y = 2x^2. \) The reduced equation has solutions of the form \( y = x^r. \) Find the general solution of the given equation.

5. Find a particular solution of \( y'' - \frac{4}{x}y' + \frac{6}{x^2}y = 6x + 2. \)

6. Find the general solution of \( y'' + 8y' + 16y = \frac{2e^{-4x}}{x^2}. \)

7. Find the general solution of \( y'' - 6y' + 9y = \frac{e^{3x}}{x}. \)

8. Find the general solution of \( y'' + 9y = 4 \cos 2x + 6x. \)

9. Find the general solution of \( y'' + 4y = 2 \sin 2x - 8. \)

10. Find the general solution of \( y'' - 6y' + 8y = 2e^{4x} + 5x - 3. \)

11. A particular solution of the nonhomogeneous differential equation

\[
y'' - 2y' - 15y = 2 \cos 3x + 5e^{5x} + 2
\]

will have the form:
12. A particular solution of the nonhomogeneous differential equation
\[ y'' - 8y' + 16y = 6e^{2x} \sin 4x + 2e^{4x} + 5x \]
will have the form:

13. The general solution of
\[ y'' + 4y' + 20y = 2xe^{-2x} + 4e^{-2x} \sin 4x + 9 \]
will have the form:

**Higher Order Linear Equations: Section 3.7**

1. The general solution of \( y''' - 4y'' + y' + 6y = 0 \) is: (Hint: \( 7e^{2x} \) is a solution.)

2. The general solution of \( y''' + y'' - 8y' - 12y = 0 \) is: (Hint: \( r = 3 \) is a root of the characteristic equation.)

3. The general solution of \( y^{(4)} + 2y''' + 4y'' - 2y' - 5y = 0 \) is: (Hint: \( e^{-x} \cos 2x \) is a solution.)

4. The homogeneous equation with constant coefficients that has
\[ y = C_1 e^{-2x} + C_2 xe^{-2x} + C_3 \cos 2x + C_4 \sin 2x + C_5 \]
as its general solution is:

5. The homogeneous equation with constant coefficients of least order that has
\[ y = 2e^{-2x} + 3 \cos 4x + 2x - 5 \]
as a solution is:

6. The homogeneous equation with constant coefficients of least order that has
\[ y = 2e^{3x} + 3e^{2x} \sin 3x + 6 \]
as a solution is:

7. A particular solution of \( y''' - 2y'' - 3y' = 2e^{-x} + xe^{3x} + 2 \) will have the form:

8. A particular solution of \( y^{(4)} - 16y = 2e^{-2x} + 3e^{4x} + \cos 2x + 5 \) will have the form:

9. The general solution of \( y^{(4)} + 5y'' - 36y = -2 \cos 3x + 3xe^{2x} \) will have the form:

10. The general solution of \( y''' + y'' + y' + y = 5 \sin x + 2e^x - e^{-x} + 4x \) will have the form:

11. A particular solution of \( y^{(4)} - 5'' - 7y'' - 3y' = 2e^x - 4xe^{3x} + 7x - 3 \) will have the form:
Laplace Transformations: Chapter 4

1. Find the Laplace transform of \( f(x) = 2e^{-3x} + \cos 2x + 5x \).

2. Find the Laplace transform of \( f(x) = 3xe^{2x} + e^x \sin 3x \).

3. If \( F(s) = \frac{2}{s^2} + \frac{s - 3}{s^2 + 1} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

4. If \( F(s) = \frac{4}{s^4 - 3s^3 + 2s^2} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

5. If \( F(s) = \frac{3s^3 + 6s^2 + 36}{s^4 + 9s^2} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

6. If \( F(s) = \frac{8}{s^2 + 2s + \frac{2s + 3}{s^2 + 16}} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

7. If \( F(s) = \frac{6s + 8}{(s - 4)(s^2 + 16)} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

8. If \( F(s) = \frac{2s + 1}{s^3 - s^2 - 8s + 12} \), then \( \mathcal{L}^{-1}[F(s)] \) is: (Hint: 2 is a root)

9. Find the Laplace transform of the solution of the initial-value problem
   \[ y' + 2y = 3 \cos 2x; \quad y(0) = 3. \]

10. Find the Laplace transform of the solution of the initial-value problem
    \[ y'' - 5y' + 6y = 4 \sin 3x; \quad y(0) = -3, \quad y'(0) = 2. \]

11. Find the Laplace transform of the solution of the initial-value problem
    \[ y'' + 25y = 2xe^{-3x}; \quad y(0) = 2, \quad y'(0) = -3. \]

12. If
    \[ f(x) = \begin{cases} 3x - 1 & 0 \leq x < 2 \\ x + 4 & x \geq 2 \end{cases} \]
    then \( \mathcal{L}[f(x)] = \)

13. If
    \[ f(x) = \begin{cases} x^2 & 0 \leq x < 3 \\ 2x + 3 & x \geq 3 \end{cases} \]
    then \( \mathcal{L}[f(x)] = \)

14. If
    \[ f(x) = \begin{cases} 4x + 4 & 0 \leq x < 1 \\ x^2 & x \geq 1 \end{cases} \]
    then \( \mathcal{L}[f(x)] = \)
15. If
\[ f(x) = \begin{cases} 
\sin x & 0 \leq x < \pi/2 \\
\cos 2x & x \geq \pi/2 
\end{cases} \]
then \( \mathcal{L}[f(x)] = \)

16. If
\[ f(x) = \begin{cases} 
x^2 + 1 & 0 \leq x < 2 \\
4e^{3x} & x \geq 2 
\end{cases} \]
then \( \mathcal{L}[f(x)] = \)

17. If \( F(s) = \frac{4}{s^2} + \frac{2}{s} + \frac{2e^{-3s}}{s^2} + \frac{4e^{-3s}}{s + 4} \), then \( \mathcal{L}^{-1}[F(s)] = f(x) \) is:

18. If \( F(s) = \frac{-2}{s^2} + \frac{4}{s} - \frac{4e^{-2s}}{s^2} + \frac{3e^{-2s}}{(s-3)^2} \), then \( \mathcal{L}^{-1}[F(s)] = f(x) \) is:

19. If \( F(s) = \frac{2s + 1}{s^2} + e^{-2s} \frac{3}{s} + 2e^{-2s} \frac{1}{s^2} \), then \( \mathcal{L}^{-1}[F(s)] = f(x) \) is:

20. If \( F(s) = \frac{s + 4e^{-3s}}{s^3 - 2s^2} \), then \( \mathcal{L}^{-1}[F(s)] = f(x) \) is:

21. If \( F(s) = \frac{4}{s} + \frac{2}{s^2} + 3e^{-2s} \frac{1}{s} - 2e^{-2s} \frac{1}{s^2} - 5e^{-4s} \frac{1}{s^2} + e^{-4s} \frac{1}{s - 2} \), then \( \mathcal{L}^{-1}[F(s)] = f(x) \) is: