INFORMATION ON EXAM 2

Content: Sections 4.1 - 4.4, 5.1 - 5.3.

This is NOT an open notes, open book exam, but you can bring to the exam an 8 1/2 x 11 sheet of paper with anything you want written on it, front and back.

I won’t ask you to regurgitate the proofs of any of the theorems in the text, but you should understand what the theorems say and what the consequences are. Some theorems are more important and/or have more “applications” than others. Try to decide what those are; see the Chapter 3 - Chapter 5 Index.

1. Statements of definitions: Complete statements like:

A sequence \((s_n)\) converges to a number \(s\) if for each number \(\epsilon > 0\), there exists an \(N\).

A sequence \((s_n)\) is a Cauchy sequence if for each number \(\epsilon > 0\), there exists an \(N\).

Let \(f : D \to \mathbb{R}\) and let \(c \in D\). Then \(f\) is continuous at \(c\) if for each \(\epsilon > 0\) there is an \(N\).

2. Statements of theorems: Complete statements like:

A sequence is convergent if and only if it is bounded.

Every bounded sequence has a limit.

If \(f : D \to \mathbb{R}\) is continuous and \(D\) is compact, then the function is uniformly continuous.

3. Questions/Problems: These will be similar to (or in some cases, the same as) the problems in the Assignments 5 – 8. Specifically:

(a) True - False questions.

(b) Calculate the limit of a given sequence, calculate subsequential limits, \(\lim\sup\) and \(\lim\inf\).

(c) Calculate \(\lim_{x \to c} f(x)\) for a given function \(f\).

(d) Use \(\epsilon - N\) or Section 4.1, Theorem 3, arguments to prove that the limit of a given sequence \(L\). Use an \(\epsilon - \delta\) argument to prove that the limit of a function is \(L\). For example, prove

\[
\lim_{x \to 2} 4x - 5 = 3 \quad \text{or} \quad \lim_{n \to \infty} \frac{3n + 2}{2n - 5} = \frac{3}{2}.
\]

(e) “Simple” proofs such as those in or similar to the homework assignments.