Nonlocal Coupling Effects on a Network of Neurons

[1] Ricardo Del Rio, [2] Shravani Deo, [3] Gabriela Jaramillo
[1]Department of Chemistry, University of Houston
[2]Department of Chemical Engineering, University of Houston
[3]Department of Mathematics, University of Houston

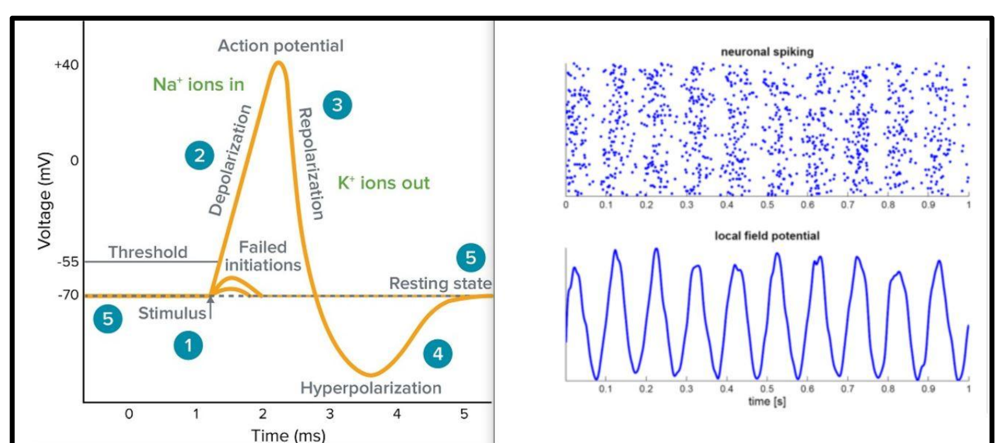
UNIVERSITY of HOUSTON

Objective

The objective of this research was to study how nonlocal coupling affects the formation of patterns and chimera states in networks of neurons.

Background

- **Systems of neurons can exhibit excitatory and oscillatory behavior. An example of excitatory behavior is neuron action potential, and an example of oscillatory behavior is the firing of neurons in a periodic manner.**
- **We use Wiener-Rosenblueth (WR) and FitzHugh-Nagumo (FHN) models to study how nonlocal coupling (the connection between neurons) affects pattern formation in excitatory and oscillatory systems, respectively.**
- **We model a network of neurons with a square grid, and apply WR and FHN equations to it.**
- **In excitatory systems, we will explore how nonlocal coupling affects spiral wave formation. In oscillatory systems, we will determine how the Levy walk model and the two layer model affect chimera formation.**



Left: neuron action potential; right: neural oscillators

Left: *What is an action potential?*, (Molecular Devices), https://www.moleculardevices.com/applications/patch-clamp-electrophysiology/what-action-potential#gref
Right: *Neural oscillation*, (Wikipedia), https://en.wikipedia.org/wiki/Neural_oscillation

Motivation

- **Experiments have found that in the brain there are mixed regions of synchronous and asynchronous neural activity. These states, called chimeras, have been found to increase efficiency in the brain [4].**
- **Research suggests that epileptic seizures occur when chimera states form in synchronous regions and then collapse [1].**
- **It is believed that stabilizing chimera states in the brain can reduce epileptic seizures.**
- **Here we study simple models describing network of neurons that are capable of reproducing chimera states. We explore the role of nonlocal coupling in stabilizing these patterns, with the hope that our results can guide research into epileptic seizures.**

FitzHugh-Nagumo Model

- **The FHN model is a simplified, dynamical, two-dimensional version of the Hodgkin-Huxley model, which models neuron behavior as an electrical circuit.**
- **FHN consists of a voltage-like variable, u, that exhibits excitatory behavior and a recovery variable, v, that provides a negative feedback.**
- **Based on the parameters, the FHN model can exhibit excitatory or oscillatory behavior. However, we will only look at the oscillatory case. This occurs when the parameter *a* in the equations below is less than 1.**
- **Using the Poincaré-Bendixson Theorem, we can prove that when *a* < 1, the system has a limit cycle and therefore displays oscillatory behavior.**

$$\epsilon\frac{du_{ij}}{dt} = u_{ij} + u_{ij}^3 - v_{ij} + \frac{\sigma}{N_r - 1}\sum_{(mn)\in B_r(i,j)} bxx(u_{ij} - u_{mn}) + bxy(v_{ij} - v_{mn})$$

$$\frac{dv_{ij}}{dt} = u_{ij} + a + \frac{\sigma}{N_r - 1}\sum_{(mn)\in B_r(i,j)} byx(u_{ij} - u_{mn}) + byy(v_{ij} - v_{mn})$$

Schmidt, Kasimatis, Hizanidis, Provata, and Hovel, *Chimera patterns in two-dimensional networks of coupled neurons*, (2017)

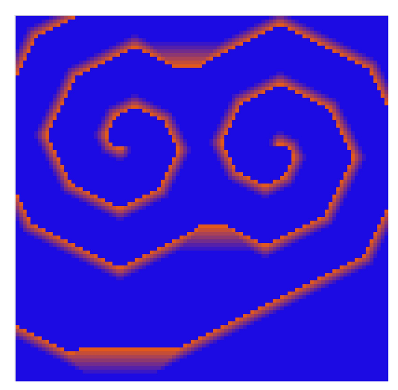
Excitatory System

Wiener-Rosenblueth Nonlocal Coupling Model

$$\Phi_{ij}^{n+1} = \begin{cases} \Phi_{ij}^n + 1, & \text{if } 0 < \Phi_{ij}^n < \tau_e + \tau_r \\ 0, & \text{if } \Phi_{ij}^n = \tau_e + \tau_r \\ 0, & \text{if } \Phi_{ij}^n = 0 \text{ and } u_{ij}^{n+1} < h \\ 1, & \text{if } \Phi_{ij}^n = 0 \text{ and } u_{ij}^{n+1} \geq h \end{cases} \qquad I_{ij}^n = \begin{cases} 1, & \text{if } 0 < \Phi_{ij}^n < \tau_e \\ 0, & \text{if } \tau_e < \Phi_{ij}^n \leq \tau_e + \tau_r \text{ or } \Phi_{ij}^n = 0; \end{cases}$$

$$u_{ij}^{n+1} = gu_i^n j + \sum_{k,l} C(k,l) I_{i+k,j+l}^n \qquad C(k,l) = \begin{cases} 1, & \text{if } |k| \leq 1 \textit{ and } |l| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- **Early research found that people's brains exhibited spiral wave like patterns when hallucinating [8].**
- **We chose to extend the coupling range of the Weiner-Rosenblueth model to see how this would affect pattern formation.**
- **Overall the spacing between spiral wave arms decreased proportionally to the range of coupling.**



τe = 5, τr = 7, g = 0, h = 3, R = 1

τe = 5, τr = 7, g = 0, h = 3, R = 3

τe = 5, τr = 7, g = 0, h = 3, R = 5

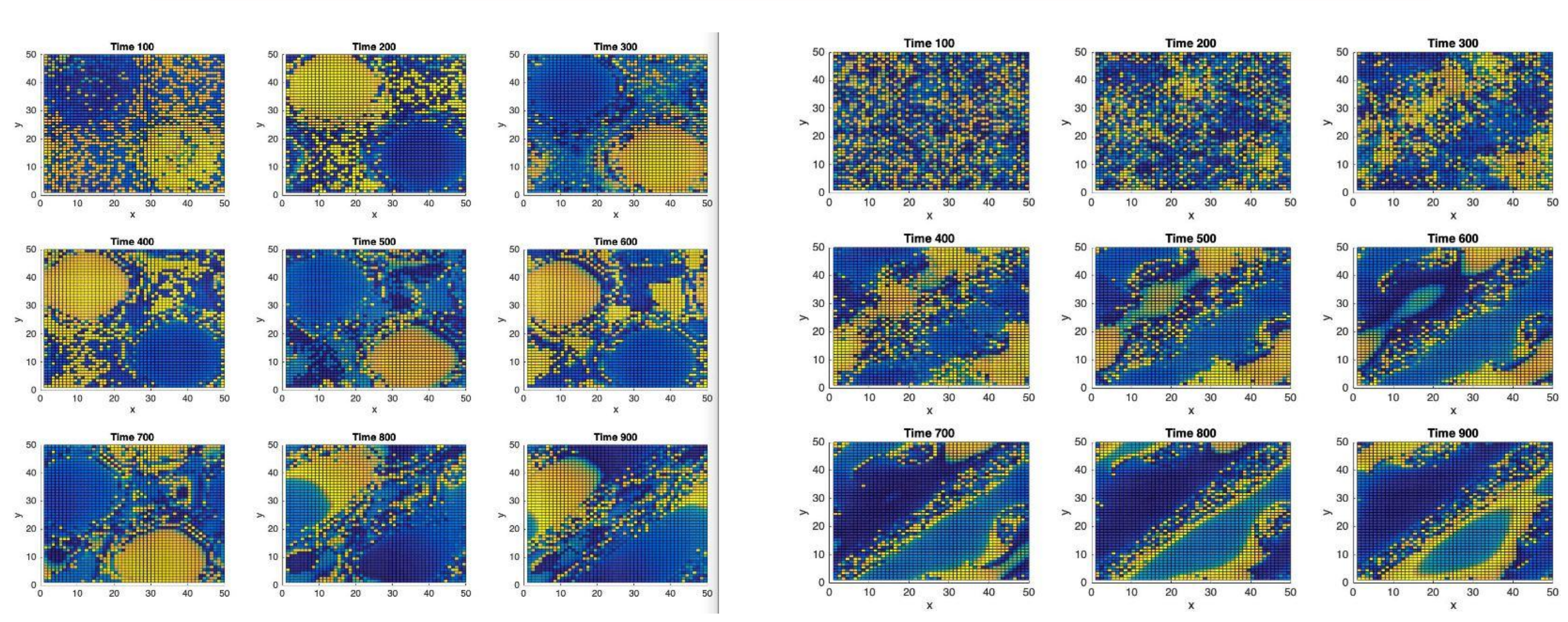
Oscillatory System

Levy-Type Coupling

- **Thinking of the steps in a random walk as connections between two different locations gives us a way for incorporating nonlocal coupling into our FHN model**
- **The Levy flights model is an extension of the random walk model used to represent foraging behavior in animals, or anomalous diffusion of molecules [7].**
- **The step-lengths follow a heavy-tailed probability distribution, which means that there is no characteristic length for the size of these random steps [7].**
- **Heavy-tailed distributions follow a power law, so long length steps are less likely than short steps. Similarly for the FHN model, neighbors that are farther away from a cell do not influence a cell as much as the neighbors that are closer.**

$$\epsilon\frac{du_{ij}}{dt} = u_{ij} + u_{ij}^3 - v_{ij} + \frac{\sigma}{N_r - 1}\sum_{(mn)\in B_r(i,j)} \frac{bxx(u_{ij} - u_{mn})}{((i-m)^2 + (j-n)^2)^{\mu} l^{\mu}} + \frac{bxy(v_{ij} - v_{mn})}{((i-m)^2 + (j-n)^2)^{\mu} l^{\mu}}$$

$$\frac{dv_{ij}}{dt} = u_{ij} + a + \frac{\sigma}{N_r - 1}\sum_{(mn)\in B_r(i,j)} \frac{byx(u_{ij} - u_{mn})}{((i-m)^2 + (j-n)^2)^{\mu} l^{\mu}} + \frac{byy(v_{ij} - v_{mn})}{((i-m)^2 + (j-n)^2)^{\mu} l^{\mu}}$$



Grid on the left: nonlocal coupling in the original FHN system; grid on the right: nonlocal coupling in the Levy flights FHN model. For both models, N = 50, a = 0.5, σ = 0.1, φ = π/2 - 0.15, r = 19, dt = 0.02.

Oscillatory System

Two Layer FHN Model

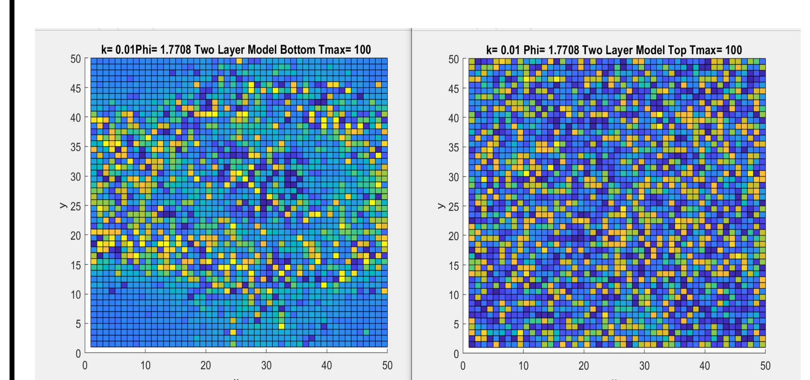
$$\epsilon\frac{du_{ij}}{dt} = u_{ij} + u_{ij}^3 - v_{ij} + \frac{\sigma}{N_r - 1}\sum_{(mn)\in B_r(i,j)} bxx(u_{ij} - u_{mn}) + bxy(v_{ij} - v_{mn}) + \kappa u_{ij}^b$$

$$\frac{dv_{ij}}{dt} = u_{ij} + a + \frac{\sigma}{N_r - 1}\sum_{(mn)\in B_r(i,j)} byx(u_{ij} - u_{mn}) + byy(v_{ij} - v_{mn}) + \kappa v_{ij}^b$$

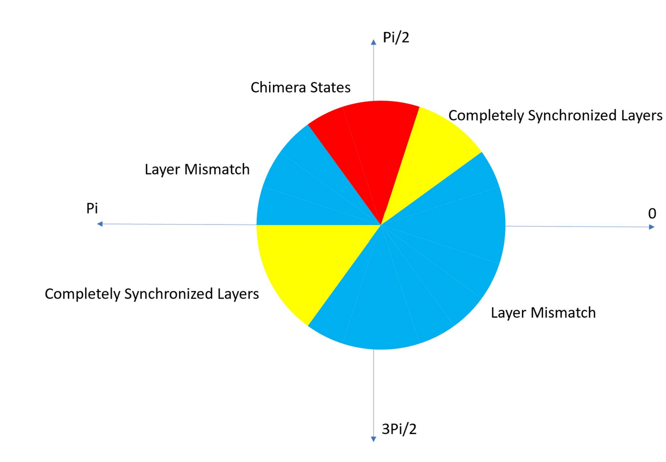
$$\epsilon\frac{du_{ij}^b}{dt} = u_{ij}^b + {u_{ij}^b}^3 - v_{ij}^b + \kappa u_{ij}$$

$$\frac{dv_{ij}^b}{dt} = u_{ij}^b + a + \kappa v_{ij}$$

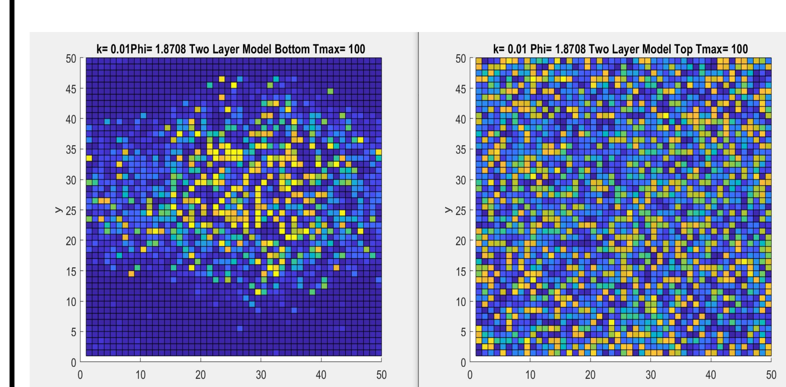
- **Recent research has found that chimera states can be synchronized in different regions of the brain using the corpus callosum as a medium(cite source).**
- **We chose to create a two layer grid of FHN oscillators which acts similar to the corpus callosum. The bottom layer has nonlocal interlayer and intralayer coupling, while the top layer contains only nonlocal interlayer coupling.**
- **We found that chimera states emerged when *φ* is approximately in the range (1.3,1.9) & *κ* =.01 in the case (R=19, N=50, σ =.10, T = 100).**



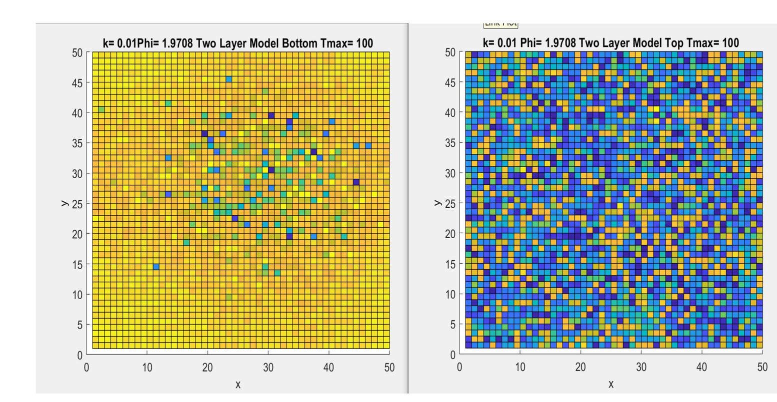
φ = 1.77, N= 50, σ=.1, R=19, T= 100, κ =.01



Different Types of Patterns observed



φ = 1.87, N= 50, σ=.1, R=19, T= 100, κ =.01



φ = 1.97, N= 50, σ=.1, R=19, T= 100, κ =.01

Conclusion/Future Works

- **In the Wiener-Rosenblueth model, nonlocal coupling changed the spacing between spiral arms.**
- **Compared to the results we got from the original FHN model, we found that Levy-type coupling changed the type of chimera patterns observed: oscillating spot chimeras were replaced by striped chimeras. Meanwhile the Two Layer FHN model was able to stabilize and create chimeras when the original FHN model could not.**
- **In the future, more cases should be tested and at longer times.**

References and Acknowledgements

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