1. Given matrices $A \in \mathbb{R}^{3 \times 5}$, $B \in \mathbb{R}^{5 \times 3}$, $C \in \mathbb{R}^{5 \times 1}$, $D \in \mathbb{R}^{3 \times 1}$. Which of the following matrix operations are allowed?
   (a) $BAD$;
   (b) $A(B + C)$;
   (c) $AC + BD$;
   (d) $ABD$.

2. Find the determinant of the matrix $A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$?

3. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$?

4. Find the inverse, assuming it exists, of the block matrix $A = \begin{bmatrix} I & 0 \\ C & B \end{bmatrix}$?

5. Consider the system of equations:
   \[
   \begin{align*}
   -3x + 4y &= 8 \\
   6x + ty &= s
   \end{align*}
   \]
   where $t$ and $s$ are real numbers.
   (a) Find values for $t$ and $s$ so that the system has exactly one solution.
   (b) Find values for $t$ and $s$ so that the system has no solutions.
   (c) Find values for $t$ and $s$ so that the system has infinitely many solutions.

6. True or False:
   (a) $U^{-T} = U$?
   (b) $(A + B)^T = B^T + A^T$?
   (c) $P^{-1} = P$ where $P$ is permutation matrix?
   (d) $(AB)^{-1} = AB$ if $A$ and $B$ are symmetric?
   (e) If $3A^2 = I$ then $A^{-1} = 3A$?
   (f) $A = \begin{bmatrix} -1 & 2 & 1 \\ -2 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ is symmetric?
   (g) If $A$ is invertible then $A^2$ and $A^{-1}$ are invertible?
   (h) If $A$ is invertible then it is necessarily square.

7. Find the inverse of $A = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$ if exists.

8. Find inverses of $E_{13}(-1) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $E_2(3) = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ if exist.
9. Find a solution to the following system:

\[
\begin{align*}
 x_1 + 2x_2 + 3x_3 + x_4 &= 4 \\
 5x_2 + 2x_3 - x_4 &= 3 \\
 3x_2 + 2x_4 &= 10
\end{align*}
\]

10. Calculate \((ABC)^T\) for the following matrices:

\[
A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 2 & 2 \\ 3 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 2 \\ 3 & -1 \end{bmatrix}.
\]

11. Transform the system:

\[
\begin{align*}
 2x + 4y - 2z &= 2 \\
 4x + 9y - 3z &= 8 \\
 -2x - 3y + 7z &= 10
\end{align*}
\]

into triangular form and construct all elimination matrices for such a transformation.

12. Construct a \(2 \times 2\) matrix \(A\) transforming \((x, y)\) into \((2x, 3y)\).

13. Solve the system:

\[
\begin{align*}
 2x + 3y &= -2 \\
 6x - y &= -6
\end{align*}
\]