DYNAMIC, TRANSIENT, MODE I CRACK PROPAGATION WITH A NONLINEAR, VISCOELASTIC COHESIVE ZONE

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ABSTRACT
The dynamic, transient propagation of a semi-infinite, mode I fracture is considered for an infinite elastic body. While the bulk constitutive response is assumed to be linearly elastic, the crack tip is modeled through inclusion of a nonlinear, viscoelastic cohesive zone. The problem is motivated by dynamic fracture in brittle polymers, while crack propagation is proceed by significant crazing in a thin region surrounding the crack tip. Homogenization of a micromechanical model of damage evolution in the form of craze growth gives rise to the nonlinear viscoelastic cohesive zone constitutive behavior utilized in the macroscale fracture initial/boundary value problem.

1 INTRODUCTION
The subject of the present contribution is unsteady, dynamic crack propagation in brittle polymers. The subject has received considerable attention in the literature, mostly focused upon experimental and numerical studies with comparatively fewer results obtained via analytical methods. Analytical solutions to select canonical fracture boundary value problems have an important role to play in gaining a deep understanding of the physical processes involved in dynamic fracture of polymeric materials and their numerical simulation. However, constructing analytical solutions to such boundary value problems, even subject to various simplifying idealizations, presents many technical obstacles.

In general, polymeric materials exhibit complex time and temperature dependent bulk constitutive behavior under both finite and infinitesimal strain histories. Moreover, fracture in such materials is nearly always preceded by craze formation as well as possibly other damage processes. Of interest here is fracture in brittle polymers (well below the glass transition temperature) which is reflected in two simplifying assumptions: (i) strains are sufficiently small to limit consideration to linearized bulk constitutive behavior; (ii) time dependent material behavior is limited to an infinitesimally thin region surrounding a fracture surface. As a consequence of these assumptions, fracture of a brittle polymer of modeled here in the context of linear elastic bulk constitutive behavior with all material time dependence confined to a crack surface cohesive zone. The cohesive zone constitutive behavior incorporates both (viscoelastic) time history and nonlinear strain dependence. The specific forms for the time dependent cohesive zone models studied below were derived through homogenization arguments applied to nonlinear, viscoelastic models of craze and damage development in front of an advancing crack in polymeric materials (Allen and Searcy [1]).

There is a growing literature devoted to constructing analytical solutions to dynamic fracture boundary value problems in the context of either linear elasticity or viscoelasticity. The treatises by (Broberg, [3]) and (Freund, [8]) give extensive account of these developments prior to 2000. Analytical solutions for dynamic steady (constant crack speed) crack growth (both steady state and transient crack growth) in linear viscoelastic material have been constructed for both mode III (anti-plane shear) (Herrmann and Walton [10], Walton [21]) and mode I (planar opening) (Walton [22], Herrmann and Walton [11]) fracture conditions.

For dynamic, unsteady crack growth, the catalog of analytical solutions in the literature is much smaller, and almost entirely confined to mode III cracks in elastic material, two exceptions being (Sarakin and Slepyan [20]) who point the way to solving dynamically accelerating mode I cracks in elastic material but do not explicitly exhibit a full solution for general loading and (Leise and Walton [16], [17]) who consider a mode III accelerating crack in a linear viscoelastic material.

A number of analytical solutions for dynamic, accelerating, mode III crack problems in the setting of linear elasticity have been constructed, both without a cohesive zone (Walton and Herrmann [24], Leise and Walton [14], [15]) and with a cohesive zone (Costanzo and Walton [6], [7]). The subject of the present contribution is to generalize this work to the setting of mode I fracture with a cohesive zone exhibiting nonlinear, history dependent constitutive behavior modeling an infinitesimally thin evolving craze field in front of an accelerating crack tip.

2 PROBLEM FORMULATION
Consider a semi-infinite crack, under pure mode I loading, with a cohesive zone of infinite extent. The problem of interest is the unsteady, dynamic growth of the crack due to tractions applied to the crack faces. The bulk material behavior is assumed to be that of an isotropic, homogeneous, linear elastic body, whereas the cohesive zone exhibits nonlinear, viscoelastic behavior, the latter being derived via homogenization of a three dimensional model of a craze/damage zone evolving in front of the advancing fracture edge (Allen and Searcy [1]).

The use of cohesive zone in modeling the fracture tip has gone in and out of favor many times since the idea was first put forward by (Barenblatt [2]). Much of the controversy surrounding their use evolves around questions of selecting cohesive zone constitutive forms that are reflective of or motivated by realistic physical processes occurring

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near a crack tip or edge. Obviously such issues are very much material or material class dependent, and since cohesive zones in brittle material are usually very small, there are serious questions of how to fit cohesive zone constitutive models to real data from dynamic fracture experiments which are very delicate and complex to carry out.

These issues notwithstanding, use of cohesive zone models is currently quite popular in direct numerical simulations of crack propagation in both the quasi-static and dynamic speed regimes. One reason for this is that cohesive zones obviate the necessity of computing stress and strain fields exhibiting singularities at fracture tips and edges. However, there is the significant issue of just how to model the cohesive zone behavior.

Barenblatt’s original conception was of a planar zone of very small length forming a transition region between fully open fracture surfaces and perfectly intact material (Figure 1a). For running cracks, the leading and trailing edges of the cohesive zone are functions of time which must be determined as part of the boundary value problem solution. Their specification makes use of the chosen cohesive zone constitutive relation and fracture criterion, AND the requirement that the stress field resulting from the cohesive zone tractions cancels the singular crack tip stress field coming from the applied loading. This latter condition can be very problematical to implement in direct numerical simulations. For that reason, (Needleman [18]) and many others have utilized cohesive zone models with cohesive zones of infinite extent. The idea is that for a semi-infinite, straight, mode I crack, say, when a load is applied, the entire line in front of the crack tip “opens up” into a cohesive zone in the sense that the displacement becomes discontinuous across a semi-infinite ray extending ahead of the crack tip. This obviates the necessity of determining the cohesive zone length required to cancel a crack tip stress singularity; only the trailing edge of the cohesive zone must be computed from a fracture criterion, such as the Critical Crack Opening Displacement.

Full specification of the cohesive zone constitutive class requires more than declaring the zone to be of either finite or infinite extent; one must also specify constitutive relations. For plane strain models of fracture under mixed mode loading (combined tensile and shear loading either on the fracture faces or far field), a commonly used class of cohesive zone models (Needleman [18], for example) has as a tunable parameter, the angle the cohesive zone traction vector makes with the crack plane. However, in general, cohesive zone models within this class violate frame indifference. This follows from a result due to (Costanzo [98]) that, unfortunately, seems not to be as well known as it should. The result establishes that a necessary condition for a cohesive zone constitutive model to be frame indifferent is that the cohesive zone traction vector and the cohesive zone opening displacement vector (i.e. the jump in the displacement across the cohesive zone) must be co-linear. In particular, if the fracture surface is planar and the loading has only a tensile component relative to the planar fracture surface (referred to here as pure mode I loading), then the cohesive traction vectors must have only a tensile component—the shear component must be zero.

2.1 THE BOUNDARY VALUE PROBLEM

The governing initial/boundary value problem is that of a semi-infinite crack propagating in an infinite linearly elastic body subject to pure mode I loading on the crack faces and with a semi-infinite cohesive zone ahead of the crack. Considerations of symmetry give the following half-plane boundary value problem.

\[
\begin{align*}
\rho \ddot{u}_i &= \sigma_{ij,j}, \quad x_2 > 0, \quad i = 1, 2 \quad \text{Momentum Equations} \\
\sigma_{ij} &= 2\mu \delta_{ij} + \delta_i \epsilon_{kk} \quad \text{Hooke’s Law} \\
u_i(x_1, x_2, 0) &= u_i(x_1, x_2, 0) = 0 \quad \text{Initial Conditions} \\
\sigma_{12}(x_1, 0, t) &= 0, \quad -\infty < x_1 < \infty \\
\sigma_{22}(x_1, 0, t) &= \sigma_{ij} \rightarrow 0 \quad \text{as } x_1^2 + x_2^2 \rightarrow \infty \\
\sigma_{ij} &= \mathcal{F}(\mathcal{Z}(x_1^2 + x_2^2)) \quad \text{Cohesive Zone Law}
\end{align*}
\]

where \(l(t)\) is the crack tip motion, \(u_i\), \(\sigma_{ij}\) are the displacement and stress components, respectively. The chosen cohesive zone law, derived in (Allen and Searcy [1]), takes the form

\[
\sigma_{ij}(x_1, t) = (1 - \alpha(t)) \left( \sigma' + \int_{t_0}^t E'(t - \tau) \frac{\partial \delta}{\partial \tau} d\tau \right)
\]

where \(\sigma'\) is the stress required for the cohesive zone to start opening, \(t_0\) is the time at which the stress attains this value, \(\alpha(t)\) is a damage parameter modeling fibril pullout and void growth within a thin craze zone, and \(\delta\) is the opening displacement nondimensionalized by the critical opening displacement at which the cohesive stresses vanish. Various forms for the viscoelastic relaxation kernel have been studied, including the standard linear solid model \(E'(t) = E_\infty + E_1 e^{-E_1 t}t^{n_1}\).

The Dirichlet-to-Neumann map corresponding to the above BVP, derived in (Saraikin and Slepyan [20]), transforms it into a boundary space/time weakly singular, nonlinear integral equation for the opening displacement. The form of the Dirichlet-to-Neumann map, found in (Saraikin and Slepyan [20]) and (Poruchikov [19]), is very long and omitted for the sake of brevity. The resulting space time boundary integral equation was solved through a delicate, combined analytical/numerical approach. It was discovered that the standard crack/cohesive zone model (Figure 1a) predicts crack face interpenetration near the trailing edge of the cohesive zone. Consequently, the BVP must be
modified to contain a contact/slip zone between the fully opened crack regime and the cohesive zone (Figure 1b). This significantly complicates solving the space/time boundary integral equation since one must now also determine the motion of the trailing edge of the contact/slip zone as well as the (compressive) stresses within that zone.

3 CONCLUSIONS

The problem considered is that of the unsteady, dynamic propagation of a semi-infinite, pure mode I crack in a brittle polymer. The bulk material is modeled as a homogeneous, isotropic linearly elastic body with the material time dependence confined to a cohesive zone ahead of the advancing crack tip. The cohesive zone constitutive behavior is modeled through a history dependent nonlinear viscoelastic like response relation incorporating an evolving damage parameter. It was shown that the classical cohesive zone paradigm of a sharp transition from fully open crack to cohesive zone (Figure 1a) must be modified in this unsteady, dynamic, mode I setting since it predicts zones of crack face interpenetration in a neighborhood of the crack tip (i.e. the trailing edge of the cohesive zone). Consequently, the classical crack/cohesive zone model must be generalized to include a contact/slip zone between the fully opened crack and the cohesive zone (Figure 1b). The extent of the contact/slip zone must be determined as part of the boundary value problem solution by imposition of the requirement that the displacement discontinuity across the fracture plane (the crack opening displacement) must be everywhere nonnegative. This effect is not seen in dynamic steady-state or transient quasi-static analyses or transient dynamic mode III analyses; it follows from properties of the Dirichlet-to-Neumann map appropriate for transient, dynamic, mode I fracture problems.

REFERENCES