Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Does addition yield a binary operation ...
   (a) (1 point) on the set \{\ldots, -4, -2, 0, 2, 4, \ldots\} of even integers? If yes, is the set with the binary operation a group?
   (b) (1 point) on the set \{\ldots, -3, -1, 1, 3, \ldots\} of odd integers? If yes, is the set with the binary operation a group?

2. In class, we defined a binary operation \(\oplus\) on \(\mathbb{Z}_n = \{0, 1, 2, \ldots, n-1\}\). We now define a binary operation \(\odot\) on \(\mathbb{Z}_n\) by setting \(a \odot b := a \cdot b\).
   (a) (1 point) Prove that \(\odot\) is associative.
   (b) (0.5 points) Does \(\mathbb{Z}_4 \setminus \{0\}\) form a group with \(\odot\)? Prove your answer.
   (c) (0.5 points) Does \(\mathbb{Z}_5 \setminus \{0\}\) form a group with \(\odot\)? Prove your answer.

3. In \(\mathbb{Z}_{13}\), solve
   (a) (1 point) the equation \(6 \oplus 9 \oplus x \oplus 2 = 7\) for \(x\).
   (b) (1 point) the equation \(7 \odot x = 5\) for \(x\).

4. (2 points) Let \((G, \ast)\) be a group such that \(x \ast x = e\) for all \(x \in G\). Prove that \(G\) is abelian.

5. (2 points) Let \((G, \ast)\) be a group. Prove that \(G\) is abelian if and only if \((x \ast y)^2 = x^2 \ast y^2\) for all \(x, y \in G\).