1. Does addition yield a binary operation ...
   (a) (1 point) on the set \{\ldots , -9, -6, -3, 0, 3, 6, 9, \ldots \} of multiples of 3? If yes, is the set with the binary operation a group?
   (b) (1 point) on the set \{\ldots , -3, -1, 1, 3, \ldots \} of odd integers? If yes, is the set with the binary operation a group?

2. (2 points) Let \( G \) be the set of all \( 2 \times 2 \) matrices
\[
\begin{pmatrix}
  a & b \\
  -b & a
\end{pmatrix},
\]
where \( a, b \in \mathbb{R} \) and \( a^2 + b^2 \neq 0 \). Prove that \( G \) forms a group with the usual matrix multiplication. You may freely use basic facts from linear algebra without proof.

3. (2 points) Let \( G \) be a group. Let \( a_1, \ldots , a_n \) be elements of \( G \). Prove that \((a_1 \ldots a_n)^{-1} = a_n^{-1} \ldots a_1^{-1}\). You must use induction to carefully prove this statement.

4. (0 points) Let \((G, \ast)\) be a group such that \( x \ast x = e \) for all \( x \in G \). Prove that \( G \) is abelian.

5. In class, we defined a binary operation \( \oplus \) on \( \mathbb{Z}_n = \{0, 1, 2, \ldots , n - 1\} \). We now define a binary operation \( \odot \) on \( \mathbb{Z}_n \) by setting \( a \odot b := a \cdot b \).
   (a) (1 point) Prove that \( \odot \) is associative.
   (b) (0.5 points) Does \( \mathbb{Z}_4 \setminus \{0\} \) form a group with \( \odot \)? Prove your answer.
   (c) (0.5 points) Does \( \mathbb{Z}_5 \setminus \{0\} \) form a group with \( \odot \)? Prove your answer.

6. In \( \mathbb{Z}_{13} \), solve
   (a) (1 point) the equation \( 2 \oplus 8 \oplus x \oplus 4 = 7 \) for \( x \).
   (b) (1 point) the equation \( 11 \odot x = 10 \) for \( x \).