1. (2 points) Find the order of $30 \in \mathbb{Z}_{54}$. Write down $\langle 30 \rangle$. You don’t need to give any proofs in your solution.

2. Let $G$ be a group.
   (a) (1 point) Prove that if $G = \langle x \rangle$, then $G = \langle x^{-1} \rangle$.
   (b) (1 point) Prove that if $G = \langle x \rangle$ and $G$ is infinite, then $x$ and $x^{-1}$ are the only generators of $G$.

3. Let $H, K$ be subgroups of a group $G$.
   (a) (1 point) Prove that $H \cap K$ is a subgroup of $G$.
   (b) (1 point) Prove that $H \cup K$ is a subgroup of $G$ if and only if $(H \subset K$ or $K \subset H)$.

4. Prove that the following sets $H$ of matrices are subgroups of $GL(2, \mathbb{R})$.
   (a) (1 point) $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + c = 1, b + d = 1, ad - bc \neq 0, a, b, c, d \in \mathbb{R} \right\}$
   (b) (1 point) $\left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a^2 + b^2 = 1, a, b \in \mathbb{R} \right\}$

5. (2 points) Let $G$ be a group and let $H$ be a nonempty subset of $G$ such that whenever $x, y \in H$, we have $x(y^{-1}) \in H$. Prove that $H$ is a subgroup of $G$. 