1. (1 point) Let \( A = \{1, 3, 5, 7, 8\} \), \( B = \{4, 5, 7\} \), \( C = \{4, 6, 7\} \). Explicitly write down the sets

\[ A \cup B \cup C, \ A \cap B \cap C, \ A \cap (B \cup C), \ B \setminus (A \cup C), \ B \setminus (A \cap C), \ A \times B. \]

2. Let \( x, y \in \mathbb{Z} \). Prove or disprove that the following relations are equivalence relations.

(a) (0.5 points) \( x \sim y \) if and only if \( x - y \) is greater than \(-1\).

(b) (0.5 points) \( x \sim y \) if and only if \( x \cdot y \leq 0 \).

(c) (0.5 points) \( x \sim y \) if and only if \( y + 7x \) is an integer multiple of \( 8 \).

3. (1 point) Let \( f : A \to B \) and \( g : B \to C \) be functions. Assume that \( f \) is injective and that \( g \circ f \) is injective. Does this imply that \( g \) is injective? Prove your answer.

4. Let the function \( f : \mathbb{Z} \to \mathbb{Z} \) be defined by

\[ f(x) = \begin{cases} 
2x + 1 & \text{if } x \text{ is even} \\
3x + 1 & \text{if } x \text{ is odd}
\end{cases} \]

(a) (1 point) Is \( f \) injective? Prove your answer.

(b) (1 point) Is \( f \) surjective? Prove your answer.

5. (1 point) Prove carefully that in any field \( F \), all \( a, b \in F \) satisfy \((-a) \cdot (-b) = a \cdot b \).

Here, for any \( x \in F \), \(-x\) denotes the unique additive inverse of \( x \).

6. (1.5 points) Prove that the set of numbers \( \{ x + y\sqrt{5} \mid x, y \in \mathbb{Q} \} \) is a field with the usual addition and multiplication of reals.

7. (1 point) Let \( z = 1 + 3i \), \( w = 1 - i \). Write \( \bar{w}, 3z - 2w, z\bar{w}, |\bar{z}|, \frac{w}{z} \) in the form \( a + bi \).

8. (1 point) Find all solutions of the equation \( z^2 - 4z + 8 = 0 \) in \( \mathbb{C} \).