UH - Math 4377/6308 - Dr. Heier - Fall 2012
HW 1
Due 09/05, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Let $A = \{1, 2, 5\}$, $B = \{4, 5\}$, $C = \{4, 6\}$. Explicitly write down the sets

$$A \cup B,\ A \cap (B \cup C),\ B \cap (A \backslash B),\ A \times C.$$

2. (3 points) Let $x, y \in \mathbb{Z}$. Prove or disprove that the following relations are equivalence relations.

(a) $x \sim y$ if and only if $x - y$ is less than 10.

(b) $x \sim y$ if and only if $x \cdot y \geq 0$.

(c) $x \sim y$ if and only if $x - y$ is even.

3. (1 point) Give an example of a set A and an relation on A which is symmetric and transitive, but not reflexive. You must not use the example of the empty relation, which has the required properties, but makes this appear more mysterious than it actually is. Hint: There is a very neat and simple one for $A = \mathbb{Z}$.

4. (3 points) Let $f : \{0, 1, 2, 3, 4\} \to \mathbb{N}, n \mapsto n^3 + n$.

(a) Find the domain, codomain and range of f.

(b) Is f one-to-one?

(c) Is f onto?

5. (1 point) Give an example of a real interval I on which the standard sin function is one-to-one with the additional property that sin is not one-to-one on any set strictly containing I. Explain your answer carefully, assuming standard facts about sin without proof.

6. (0.5 points) Let $z = 1 + 3i$, $w = 1 - 2i$. Write $\bar{z}$, $z + w$, zw, $|z|$, $\frac{1}{z}$ in the form $a + bi$.

7. (0.5 points) Solve $z^2 - 4z + 13 = 0$ in $\mathbb{C}$.

8. **(1 extra credit point)** Let $x, y \in \mathbb{Z}$. Let $x \sim y$ if and only if $y + 4x$ is an integer multiple of 5. Prove that $\sim$ is an equivalence relation.