UH - Math 4377/6308 - Dr. Heier - Fall 2017
HW 1
Revised due date: 09/07, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Let $A = \{1, 2, 5\}$, $B = \{4, 5\}$, $C = \{4, 6\}$. Explicitly write down the sets

$$A \cup B,\ A \cap B,\ A \cap (B \cup C),\ A \cap B \cap C,\ A \times C.$$

2. Let $x, y \in \mathbb{Z}$. Prove or disprove that the following relations are equivalence relations.

(a) (0.5 points) $x \sim y$ if and only if $x - y$ is less than 10.

(b) (0.5 points) $x \sim y$ if and only if $x \cdot y \geq 0$.

(c) (0.5 points) $x \sim y$ if and only if $x - y$ is even.

3. Let $f : A \to B$ and $g : B \to C$ be functions.

(a) (1 point) Assume that $g \circ f$ is injective. Does this imply that both f and g are injective? Prove your answer.

(b) (1 point) Assume that $g \circ f$ is surjective. Does this imply that both f and g are surjective? Prove your answer.

4. Let $f : \{0, 1, 2, 3, 4\} \to \mathbb{N}, n \mapsto n^3 + n$.

(a) (0.5 points) Find the domain, codomain and range of f.

(b) (0.5 points) Is f one-to-one? Prove your answer.

(c) (0.5 points) Is f onto? Prove your answer.

4. (1 point) Prove carefully that in any field F, all $a, b \in F$ satisfy $a \cdot (-b) = -(a \cdot b)$. Here, for any $x \in F$, $-x$ denotes the unique additive inverse of x.

5. (1 point) Prove that the set of numbers $\{x + y\sqrt{3} \,|\, x, y \in \mathbb{Q}\}$ is a field with the usual addition and multiplication of reals.

6. (1 point) Let $z = 1 + 3i$, $w = 1 - 2i$. Write $\bar{z}$, $z + w$, zw, $|z|$, $\frac{1}{z}$ in the form $a + bi$.

7. (1 point) Find all solutions of the equation $z^2 - 2z + 5 = 0$ in $\mathbb{C}$.

8. (1 extra credit point) Let $x, y \in \mathbb{Z}$. Let $x \sim y$ if and only if $y + 4x$ is an integer multiple of 5. Prove that $\sim$ is an equivalence relation.