UH - Math 6302/Modern Algebra - Dr. Heier - Fall 2018
HW 1
Due Wednesday, Sep. 19, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Let G be a group. Assume that $g^2 = e$ holds for all $g \in G$. Prove that G is commutative.

2. (1 point) Let G be a group and $H \subset G$ a finite non-empty subset. Prove that H is a subgroup of G if and only if $\forall\ x, y \in H : xy \in H$.

3. (1 point) Let G be a group of finite order $n > 2$. Prove that there does not exist a subgroup of order $n-1$ in G. Note: You must NOT use Lagrange's Theorem. Give a direct proof.

4. (1 point) Let G be an infinite cyclic group. Prove that G is isomorphic to $\mathbb{Z}$.

5. (1 point) Draw the "lattice of subgroups" for the symmetric group S_3.

6.

(a) (1 point) Let H be a subgroup of the group G. Prove that H is a subgroup of the normalizer $N_G(H)$.

(b) (1 point) Give an example to show that if H is merely a subset of G, then H may not be a subset of $N_G(H)$.

7. (1 point) Let $\varphi : G \to H$ be a homomorphism of groups. Prove that $\ker \varphi$ is a normal subgroup of G. Prove that $\operatorname{im} \varphi$ is a subgroup of H. Is $\operatorname{im} \varphi$ always normal? Prove your answer.

8. (2 points) Prove that if $G/Z(G)$ is cyclic, then G is commutative.