UH - Math 3336 - Dr. Heier - Fall 2019
HW 2
Due Thursday, 09/05, at the beginning of class.
Solutions may be handwritten. Use regular sheets of paper, stapled together.
Do not forget to write your name on page 1.

1. Determine whether the following compound propositions are satisfiable.

(a) (0.5 points) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

(b) (0.5 points) $(p \Rightarrow q) \wedge (p \Rightarrow \neg q) \wedge (\neg p \Rightarrow q) \wedge (\neg p \Rightarrow \neg q)$

(c) (0.5 points) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$

(d) (0.5 points) $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg s) \wedge (\neg p \vee \neg r \vee \neg s) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg r \vee \neg s)$

2. Determine the truth value of each of the following statements if the domain consists of all integers, i.e., the set $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$.

(a) (0.5 points) $\exists x(2x = x^2)$

(b) (0.5 points) $\exists x(x > 2x)$

(c) (0.5 points) $\forall x(x \leq 2x)$

(d) (0.5 points) $\forall x(x \leq x^3)$

3. Suppose that the domain of the propositional function $P(x)$ consists of the integers $1, 2, 3, 4$. Express the following statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

(a) (0.25 points) $\exists x P(x)$

(b) (0.25 points) $\forall x P(x)$

(c) (0.5 points) $\neg \exists x P(x)$

(d) (0.5 points) $\neg \forall x P(x)$

(e) (0.5 points) $\forall x((x > 1) \Rightarrow P(x)) \vee \exists x \neg P(x)$

4. Express the negation of each of the following statements in terms of quantifiers without using the negation symbol.

(a) (0.5 points) $\forall x(-1 \leq x < 5)$

(b) (0.5 points) $\forall x(-2 < x < 7)$

(c) (0.5 points) $\exists x(0 \leq x \leq 1)$

(d) (0.5 points) $\exists x(0 < x \leq 2)$

5. Rewrite each of these statements so that negations appear only within predicates, i.e., so that no negation is outside of a quantifier or an expression involving logical connectives.

(a) (1 point) $\neg \exists y(Q(y) \wedge \forall x \neg R(x, y))$

(b) (1 point) $\neg \exists y(\forall x Q(x, y) \vee \exists x R(x, y))$