UH - Math 4377 - Dr. Heier - Spring 2010
HW 2 – due 02/04 at the beginning of class

1. Does

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2) \text{ and } c(a_1, a_2) = (ca_1, a_2)$$

define a vector space structure on the 2-tuples of real numbers? Justify your answer.

2. Prove that in a vector space V, the zero vector $\vec{0}$ is unique.

3. Determine if the following subsets of $\mathbb{R}^3$ are subspaces.

(a) $\{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 - 3a_3 = 0\}$

(b) $\{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 - 2a_2 + a_3 = 1\}$

(c) $\{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 = a_3\}$

(d) $\{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 = 5a_3 \text{ and } 4a_2 = a_1 + a_3\}$

4. Determine if the following subsets of the vector space of 2×2 matrices with real entries are subspaces. You may assume as true that the set of 2×2 matrices with real entries forms a vector space with the usual addition and scalar multiplication.

(a) $\left\{ \begin{pmatrix} a_1 & a_2 \\ a_3 & 0 \end{pmatrix} : a_1, a_2, a_3 \in \mathbb{R} \right\}$

(b) $\left\{ \begin{pmatrix} a_1 & a_2 \\ a_2 & a_3 \end{pmatrix} : a_1, a_2, a_3 \in \mathbb{R} \right\}$

5. A real-valued function f defined on the real line is called an *even function* if $f(t) = f(-t)$ for each real number t. Prove that the set of even functions is a subspace with the usual addition and scalar multiplication for functions. You may assume as true that the set of real-valued functions f defined on the real line is a vector space with the usual addition and scalar multiplication for functions.

6. Let W_1, W_2 be two subspaces of a vector space V. Prove that the intersection $W_1 \cap W_2$ is also a subspace of V.

7. (extra credit) Let W_1, W_2 be two subspaces of a vector space V. Prove that the union $W_1 \cup W_2$ is a subspace of V if and only if $W_2 \subseteq W_1$ or $W_1 \subseteq W_2$.