UH - Math 3330 - Dr. Heier - Spring 2019
HW 10
Due Wednesday, 04/17, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. (2 points) Consider $(\mathbb{Q}, \oplus, \odot)$, where $\oplus$ is the addition given by $a \oplus b = a + b - 1$ and $\odot$ is the multiplication given by $a \odot b = a + b - ab$. Is $(\mathbb{Q}, \oplus, \odot)$ a field?

2. An element r in a ring is called *idempotent* if $r^2 = r$. Let R be a ring with unity 1. Let $r \in R$ be idempotent. Prove that

(a) (1 point) $1 - r$ is also idempotent;

(b) (1 point) r or $1 - r$ is a zero-divisor.

3. Let R be a ring with unity. Then R is called *Boolean* if every element of R is idempotent. Prove that if R is Boolean, then

(a) (1 point) $r = -r$ for every $r \in R$;

(b) (1 point) R is commutative.

4. (2 points) Let R be a ring with unity. Assume that R has no non-zero zero-divisors. Let $a, b \in R$ with $ab = 1$. Prove that $ba = 1$.

5. (2 points) Let F be the set of all 2×2 matrices of real numbers of the form

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix}.$$

Prove that F forms a field under the usual addition and multiplication of matrices.