UH - Math 6303 - Dr. Heier - Spring 2019
HW 4
Due Friday, 05/03, 1pm, in my departmental mailbox or as email attachment.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Let k be a finite field. Prove that every subset of $\mathbb{A}^n$ is an affine algebraic set.

2. (1 point) Let k be a field. Identify the 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with entries in k with the point (a, b, c, d) in $\mathbb{A}^4$. Show that the group $SL_2(k)$ of matrices of determinant 1 is an algebraic set in $\mathbb{A}^4$.

3. (2 points) Let $V = Z(xy - z) \subset \mathbb{A}^3$. Prove that V is isomorphic to $\mathbb{A}^2$ and provide an explicit isomorphism φ and associated k-algebra isomorphism $\tilde{\varphi} : k[V] \to k[\mathbb{A}^2]$, along with their inverses.

4. (1 point) Prove that $GL_n(k)$ is a Zariski-open subset of $\mathbb{A}^{n^2}$. Also, prove that the elements of $GL_n(k)$ are in bijective correspondence with the points of an affine algebraic set in $\mathbb{A}^{n^2+1}$.

5. (1 point) Let $\varphi : V \to W$ be a surjective morphism of affine algebraic sets. Prove that if V is an affine variety, then W is an affine variety.

6. (1 point) Give an example of an integral domain which is not normal. You don't need to prove that your ring is an integral domain, but you do need to prove it is not normal.

7. (1 point) Let k be a field. Let $a_1, \ldots, a_n \in k$. Prove that $(x_1 - a_1, \ldots, x_n - a_n)$ is a maximal ideal in $k[x_1, \ldots, x_n]$.

8. (2 points) Let k be an algebraically closed field. Prove that every proper radical ideal in $k[x_1, \ldots, x_n]$ is an intersection of maximal ideals.