UH - Math 3330 - Dr. Heier - Spring 2020
HW 5
Due Thursday, 02/20, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Calculate the order of

(a) (1 point) $(4, 9)$ in $\mathbb{Z}_5 \times \mathbb{Z}_{15}$

(b) (1 point) $(1, 8)$ in $\mathbb{Z}_2 \times \mathbb{Z}_{22}$

2. Let $f : A \to B$ be a function between non-empty sets A, B.

(a) (1 point) Prove that f is injective if and only if there exists a function $g : B \to A$ such that $g \circ f = \mathrm{id}_A$, where $\mathrm{id}_A : A \to A, a \mapsto a$ is the identity function.

(b) (1 point) Prove that f is surjective if and only if there exists a function $g : B \to A$ such that $f \circ g = \mathrm{id}_B$.

3. Let $f : A \to B$ and $g : B \to C$ be functions.

(a) (1 point) Assume that $g \circ f$ is injective. Does this imply that both f and g are injective? Prove your answer.

(b) (1 point) Assume that $g \circ f$ is surjective. Does this imply that both f and g are surjective? Prove your answer.

4. Execute the following multiplications in S_7.

(a) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 7 & 1 & 3 & 2 & 6 & 4 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 1 & 3 & 5 & 7 & 4 & 2 \end{pmatrix}$.

(b) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 5 & 7 & 4 & 6 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 5 & 6 & 4 & 2 & 3 & 1 \end{pmatrix}$.

5. Write each of the following permutations as a product of disjoint cycles and then as a product of transpositions. Determine whether each permutation is odd or even.

(a) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 4 & 3 & 9 & 6 & 7 & 8 & 5 & 1 & 10 & 2 \end{pmatrix}$.

(b) (1 point) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 3 & 1 & 6 & 4 & 5 & 8 & 9 & 10 & 2 \end{pmatrix}$.