Review for Exam 3
Calculus I (invitation-only)

This serves as a review of particular concepts and examples that "should" help prepare you for your upcoming exam. It is NOT a full description of things that may appear on the exam!! You should also follow Dr. Climenhaga's review sheet and review class notes, homework assignments, and examples from lecture and/or lab.

Definitions

1. Explain the difference between an absolute maximum and a local maximum.

2. (a) Give the definition of a critical number for a function f.

   (b) Give the definition of a point of inflection for a function f.

3. Define what it means for a function f to be increasing and decreasing in terms of its derivative.

4. State the Second Derivative Test.

5. Give the formal definitions and an illustration of what it means for on a given interval I a function f is concave and convex.

6. What is an antiderivative of a function f?

7. Define the definite integral of a function f from a to b in full detail. Also, mention what it means for f to be integrable.

Computational Techniques

1. Find the absolute extrema of $f(x) = x^3 + 3x^2 - 1$ on the interval $[-1, 2]$.

2. Find the absolute extrema of $f(x) = e^{-x^2}$ on the interval $[-1, 1]$.

3. Given $f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - 2x^2 - 8x + 4$, find any local extrema using the first derivative test.

4. Given $f(x) = x^2 e^{-x}$, find any local extrema using the second derivative test.

5. Given $f(x) = x^4 - 2x^3 + 6$, use the first derivative analysis of f to find critical points, intervals of increase/decrease, and local maxima/minima.

6. Given $f(x) = \frac{5x}{(x-1)^2}$, use the second derivative analysis of f to find intervals of convexity/concavity and any points of infection.

7. Evaluate the limits.

   (a) $\lim_{x\to 0} \frac{e^x - 1}{\tan x}$

   (b) $\lim_{x\to -\infty} (x^2 - x^3)e^{2x}$

   (c) $\lim_{x\to(\pi/2)^-} (\tan x)^{\cos x}$

   (d) $\lim_{x\to 1^+} (\frac{x}{x-1} - \frac{1}{\ln x})$

8. Find the antiderivative of $f(x) = \sqrt[3]{x^2} - x\sqrt{x}$.

9. Find the antiderivative of $r(\theta) = \sec x \tan x - 2e^x$.

10. Given $f'(x) = 5x^4 - 3x^2 + 4$ with initial condition $f(-1) = 2$, find f.

11. Find the area under the graph of $f(x) = x^2 + \sqrt{1 + 2x}$ on $4 \le x \le 7$ as a limit.

12. Use Theorem 5.2.4 to evaluate the integral $\int_0^2 (2x - x^3)\ dx$.

13. Problem 5.2.34 from the text book.

14. Evaluate $\int_0^1 \left|2x - 1\right|\ dx$ by interpreting it in terms of area.

15. Evaluate $\int_0^1 (1 - 8v^3 + 16v^7)\ dv$.

16. Evaluate $\int_1^4 \frac{2+x^2}{\sqrt{x}}\ dx$.

17. Evaluate $\int_0^3 (2\sin x - e^x)\ dx$.

Applications

1. Find the point on the line $y = 2x + 3$ that is closest to the origin.

2. A farmer wants to fence in an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?

3. Find the dimensions of the rectangle of largest are that can be inscribed in a circle of radius r.

Theoretical Results

1. (a) State Rolle's Theorem.

   (b) State the Mean Value Theorem and give a geometric interpretation.

2. State l'Hopital's Rule.