Selected Homework Solutions

Homework 11

Section 5.4

#33) Evaluate $\int_1^2 \left(\frac{x}{2} - \frac{2}{x}\right) dx$.

$$\int_1^2 \left(\frac{x}{2} - \frac{2}{x}\right) dx = \frac{1}{2}\int_1^2 x\, dx - 2\int_1^2 \frac{1}{x}\, dx$$

$$= \frac{1}{2}\cdot\left(\frac{x^2}{2}\Big|_1^2\right) - 2\left(\ln x \Big|_1^2\right)$$

$$= \frac{1}{2}\left(\frac{4}{2} - \frac{1}{2}\right) - 2\left(\ln 2 - \cancel{\ln 1}\right)$$

$$= \frac{1}{2}\left(\frac{3}{2}\right) - 2\ln 2 = \boxed{\frac{3}{4} - 2\ln 2}$$

#39) Evaluate $\int_1^8 \frac{2+t}{\sqrt[3]{t^2}}\, dt$.

$$\int_1^8 \frac{2+t}{\sqrt[3]{t^2}}\, dt = \int_1^8 (2+t)\, t^{-2/3}\, dt = \int_1^8 2t^{-2/3} + t^{1/3}\, dt$$

$$= \left(2\cdot 3t^{1/3} + \frac{3}{4}t^{4/3}\right)\Big|_1^8$$

$$= 6(8)^{1/3} + \frac{3}{4}(8)^{4/3} - \left(6(1) + \frac{3}{4}(1)\right)$$

$$= 12 + \frac{3}{4}\cdot 16 - 6 - \frac{3}{4}$$

$$= 18 - \frac{3}{4} = \boxed{\frac{69}{4}}$$

Section 5.5

#27) Evaluate $\int (x^2+1)(x^3+3x)^4\,dx$.

Let $u = x^3+3x$. Then $du = (3x^2+3)\,dx \Rightarrow \frac{du}{3} = (x^2+1)\,dx$.

$$\therefore \int (x^2+1)(x^3+3x)^4\,dx = \frac{1}{3}\int u^4\,du = \frac{1}{3}\cdot\frac{u^5}{5} + C$$
$$= \boxed{\frac{1}{15}(x^3+3x)^5 + C}$$

#66) Evaluate $\int_{-\pi/3}^{\pi/3} x^4 \sin x\,dx$.

Recall that $\sin(-x) = -\sin x$. Thus if $f(x) = x^4\sin x$, $f(-x) = (-x)^4\sin(-x) = -x^4\sin x = -f(x)$. Hence, f is odd. Therefore, $\int_{-\pi/3}^{\pi/3} x^4\sin x\,dx = 0$.

Section 6.1

\# 18) Sketch the region enclosed by $y=\sqrt{x-1}$ and $x-y=1$. Find the area of the region.

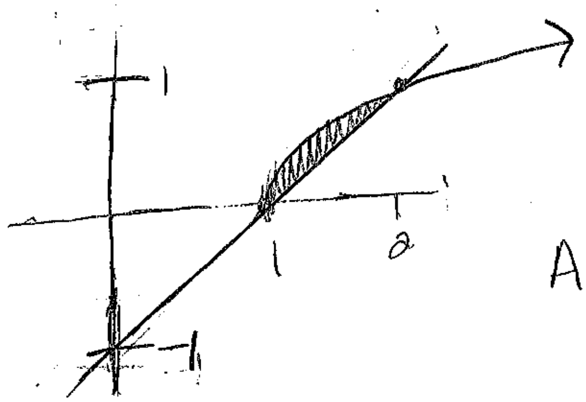
$y = x-1$

$$x-1=\sqrt{x-1} \Rightarrow (x-1)^2 = x-1$$

$$x^2-2x+1 = x-1$$

$$x^2-3x+2=0$$

$$(x-1)(x-2)=0 \Rightarrow x=1,2$$



$$A_R = \int_1^2 \left[\sqrt{x-1} - (x-1)\right] dx$$

$$= \int_1^2 \sqrt{x-1}\, dx - \int_1^2 x-1\, dx$$

$$= \int_0^1 u^{1/2}\, du - \int_1^2 x-1\, dx$$

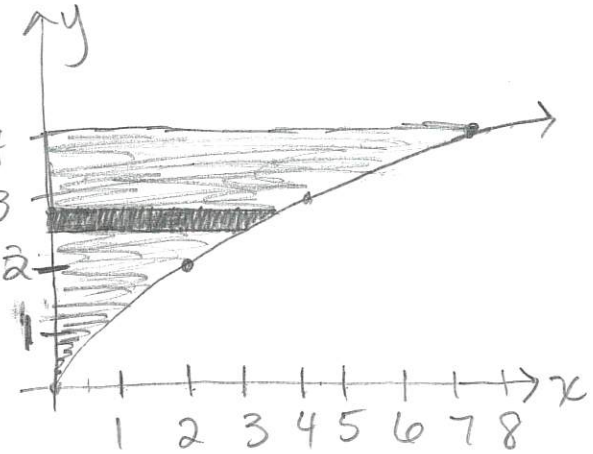
$$= \frac{2u^{3/2}}{3}\Big|_0^1 - \left(\frac{x^2}{2} - x \Big|_1^2\right)$$

$$= \frac{2}{3} - \left(\cancel{2-2} - \left(\frac{1}{2}-1\right)\right) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

Section 6.2

#6) Find the volume of the solid obtained by rotating the region bounded by $2x = y^2$, $x = 0$, $y = 4$ about the y-axis.

$x = 0 \Rightarrow y = 0$



$A(y) = \pi x^2 = \pi \left(\frac{1}{2}y^2\right)^2 = \frac{\pi}{4} y^4$

$V = \int_0^4 \frac{\pi}{4} y^4 \, dy = \frac{\pi}{4}\left[\frac{y^5}{5}\Big|_0^4\right]$

$= \frac{\pi}{20} \cdot 4^5 = \frac{256\pi}{5}$