Section 3.4 #31, 41; Section 3.5 # 12, 19, 20
Section 3.6 # 43, 45, 47; Section 3.8 #3, 8, 9
Section 3.9 #25, 33

Section 3.4

#31 Differentiate $F(t) = e^{t\sin 2t}$.

$$F'(t) = e^{t\sin 2t} \cdot (\sin 2t + 2t\cos 2t)$$

#41 Differentiate $f(t) = \sin^2(e^{\sin^2 t})$.

$$f'(t) = 2\sin(e^{\sin^2 t})\cos(e^{\sin^2 t}) \cdot e^{\sin^2 t} \cdot 2\sin t\cos t$$

Section 3.5

#12 Find $\frac{dy}{dx}$ of $\cos(xy) = 1 + \sin y$.

$$-\sin(xy)\left(y + x\frac{dy}{dx}\right) = \cos y\frac{dy}{dx}$$

$$-y\sin(xy) = (\cos(y) + x\sin(xy))\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-y\sin(xy)}{\cos(y) + x\sin(xy)}$$

#19 Find $\frac{dy}{dx}$ of $\sin(xy) = \cos(x+y)$.

$$\cos(xy)\left(y + x\frac{dy}{dx}\right) = -\sin(x+y)\left(1 + \frac{dy}{dx}\right)$$

$$y\cos(xy) + \sin(x+y) = -(\sin(x+y) + x\cos(xy))\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y\cos(xy) + \sin(x+y)}{-(\sin(x+y) + x\cos(xy))}$$

#20 Find $\frac{dy}{dx}$ of $\tan(x-y) = \frac{y}{1+x^2}$

$$\sec^2(x-y)\left(1-\frac{dy}{dx}\right) = \frac{(1+x^2)\frac{dy}{dx} - 2yx}{(1+x^2)^2}$$

$$(1+x^2)^2\sec^2(x-y)\left(1-\frac{dy}{dx}\right) = (1+x^2)\frac{dy}{dx} - 2xy$$

$$(1+x^2)^2\sec^2(x-y) + 2xy = (1+x^2)(1+(1+x^2)\sec^2(x-y))\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{(1+x^2)^2\sec^2(x-y) + 2xy}{(1+x^2)(1+(1+x^2)\sec^2(x-y)}$$

Section 3.6

Use logarithmic differentiation to differentiate

#43 $y = x^x$

$$\ln y = x\ln x$$

$$\frac{y'}{y} = \ln x + x\cdot\frac{1}{x} = \ln x + 1$$

$$y' = x^x(\ln x + 1)$$

#45 $y = x^{\sin x}$

$$\ln y = \sin x \cdot \ln x$$

$$\frac{y'}{y} = \cos x \ln x + \frac{\sin x}{x}$$

$$y' = x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$$

#47 $y = (\cos x)^x$

$$\ln y = x\ln\cos x$$

$$\frac{y'}{y} = \ln\cos x - \frac{x\sin x}{\cos x}$$

$$y' = (\cos x)^x(\ln\cos x - x\tan x)$$

Section 3.8

#3 A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420.

a) Find an expression for the number of bacteria after t hours.

$y(t) = y_0 e^{kt} = 100e^{kt}$

$y(1) = 420 = 100e^{k}$

$4.2 = e^{k} \Rightarrow \ln 4.2 = k$

$\therefore y(t) = 100e^{\ln 4.2\, t} = 100(e^{\ln 4.2})^{t} = 100(4.2)^{t}$

b) Find the number of bacteria after 3 hours.

$y(3) = 100(4.2)^3 = 7408.8 \approx 7409$

c) Find the rate of growth after 3 hours.

$y'(t) = 100 \cdot (4.2)^{t} \cdot \ln 4.2$

$y'(3) = 100 \cdot \ln 4.2 \cdot (4.2)^3 \approx 10632$ bacteria/hour

d) When will the population reach 10,000?

$10000 = 100(4.2)^{t}$

$100 = 4.2^{t}$

$\frac{\ln 100}{\ln 4.2} = t \approx 3.2$ hours

#8 Strontium-90 has a half-life of 28 days.

a) A sample of mass 50mg initially. Find a formula for the mass remaining after t days.

$y(t) = y_0 e^{kt} = 50e^{kt}$

$25 = 50e^{28k}$

$\ln \frac{1}{2} = 28k \qquad \frac{-\ln 2}{28} = k$

$\therefore y(t) = 50e^{-\frac{\ln 2}{28}t} = 50(2)^{-t/28}$

b) Find the mass remaining after 40 days.

$y(40) = 50 \cdot 2^{-40/28} = 50 \cdot 2^{-10/7} \approx 19$

c) How long does it take the sample to decay to 2mg?

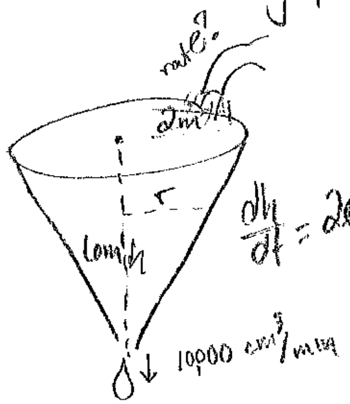
$2 = 50 \cdot 2^{-t/28}$

$\ln \frac{1}{25} = -\frac{t}{28}\ln 2$

$28 \ln 25 = t \ln 2 \Rightarrow t = \frac{28 \ln 25}{\ln 2} = 130$ days

Section 3.9

#25 Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4m. If the water level is raising at a rate of 20 cm/min when the height is 2m, find the rate at which water is being pumped into the tank.



$\frac{dh}{dt} = 20\ cm/min$

rate out: 10000 cm^3/min rate in: ? x

$$\frac{dV}{dt} = x - 10000 = \text{rate in} - \text{rate out}$$

$$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3}\left(\frac{h}{3}\right)^2 h = \frac{\pi}{27} h^3$$

(Similar triangles $\Rightarrow \frac{r}{h} = \frac{200}{600} \Rightarrow r = \frac{h}{3}$)

2m = 200 cm

6m = 600 cm

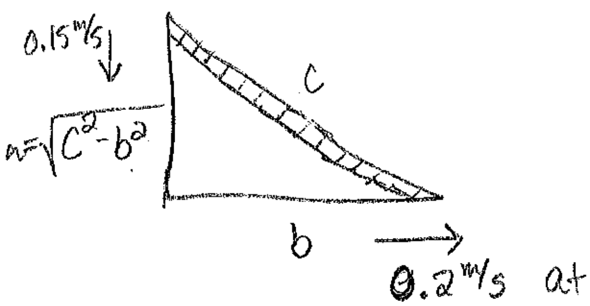
$$\Rightarrow \frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$$

$$x - 10000 = \frac{\pi}{9}(200)^2(20)$$

$$x = \frac{20\pi}{9}(200)^2 + 10000 = 289{,}252.7\ cm^3/min$$

$$= 2.89 \times 10^5\ cm^3/min$$

#33 The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder?



$$a^2 + b^2 = c^2$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$a\frac{da}{dt} + b\frac{db}{dt} = c\frac{dc}{dt}$$

$$(\sqrt{c^2-9})(-0.15) + 3(0.2) = 0$$

$$\sqrt{c^2-9} = +\frac{0.6}{0.15} = 4$$

$$c^2 - 9 = 16$$

$$c^2 = 25$$

$$\boxed{c = 5\,m}$$

$$\frac{da}{dt} = -0.15$$

$$\frac{db}{dt} = 0.2$$

$$\frac{dc}{dt} = 0$$

$$a = \sqrt{c^2 - b^2}$$

$$b = 3$$