Solutions to Selected Problems

Homework 1

Section 7.1

21) Evaluate $\int \frac{xe^{2x}}{(1+2x)^2}\,dx$.

Let $u = xe^{2x}$ and $dv = \frac{dx}{(1+2x)^2}$ $\quad w = 1+2x$, $dw = 2\,dx$

$du = (e^{2x} + 2xe^{2x})\,dx$ $\quad \Rightarrow v = -\frac{1}{2}\cdot\frac{1}{1+2x}$

$= (1+2x)e^{2x}\,dx$

$$\therefore \int \frac{xe^{2x}}{(1+2x)^2}\,dx = -\frac{1}{2}\cdot\frac{xe^{2x}}{1+2x} + \frac{1}{2}\int \frac{1}{\cancel{1+2x}}\cdot\cancel{(1+2x)}e^{2x}\,dx$$

$$= -\frac{xe^{2x}}{2(1+2x)} + \frac{1}{2}\int e^{2x}\,dx$$

$$= -\frac{xe^{2x}}{2(1+2x)} + \frac{1}{4}e^{2x} + C$$

$$= \frac{e^{2x}}{8x+4} + C$$

33) Evaluate $\int_0^{\pi/3} \sin x \ln(\cos x)\,dx$.

Let $u = \cos x$. Then $du = -\sin x\,dx$. Thus,

$$\int_0^{\pi/3} \sin x \ln(\cos x)\,dx = \int_1^{1/2} -\ln u\,du = \int_{1/2}^1 \ln u\,du.$$

Now let $w = \ln u$ and $dv = du$.

$dw = \frac{du}{u}$ $\quad v = u$

$$\therefore \int_{1/2}^1 \ln u\,du = u\ln u\Big|_{1/2}^1 - \int_{1/2}^1 u\cdot\frac{du}{u} = u\ln u\Big|_{1/2}^1 - \int_{1/2}^1 du$$

$$= u\ln u\Big|_{1/2}^1 - u\Big|_{1/2}^1$$

$$= \cancel{1\cdot\ln 1} - \tfrac{1}{2}\ln\tfrac{1}{2} - 1 + \tfrac{1}{2}$$

$$= \tfrac{1}{2}(1 - \ln\tfrac{1}{2}) - 1 = -\tfrac{1}{2}(\ln\tfrac{1}{2} + 1)$$

#42) Evaluate the integral $\int \frac{\arcsin(\ln x)}{x}\,dx$.

Let $w = \ln x$. Then $dw = \frac{1}{x}dx$. Therefore,

$$\int \frac{\arcsin(\ln x)}{x}\,dx = \int \arcsin(w)\,dw$$

$u = \arcsin(w)$ $\quad v = w$

$du = \frac{dw}{\sqrt{1-w^2}}$ $\quad dv = dw$

$$= w\arcsin(w) - \int \frac{w\,dw}{\sqrt{1-w^2}}$$

Now let $u = 1-w^2$, then $du = -2w\,dw$. Thus,

$$\int \frac{\arcsin(\ln x)}{x}\,dx = w\arcsin(w) + \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$$

$$= w\arcsin(w) + u^{1/2} + C$$

$$= \ln x \cdot \arcsin(\ln x) + \sqrt{1-(\ln x)^2} + C$$

Section 7.2

#6) Evaluate $\int t\cos^5(t^2)\,dt$.

Let $u = t^2$, then $du = 2t\,dt$. Therefore,

$$\int t\cos^5(t^2)\,dt = \frac{1}{2}\int \cos^5(u)\,du$$

$$= \frac{1}{2}\int (\cos^2(u))^2 \cos(u)\,du$$

$$= \frac{1}{2}\int (1-\sin^2(u))^2 \cdot \cos(u)\,du$$

Now let $w = \sin(u)$, then $dw = \cos(u)\,du$. Thus,

$$\int t\cos^5(t^2)\,dt = \frac{1}{2}\int (1-\sin^2(u))^2 \cdot \cos(u)\,du$$

$$= \frac{1}{2}\int (1-w^2)^2\,dw = \frac{1}{2}\int (1-2w^2+w^4)\,dw$$

$$\boxed{\frac{1}{2}\left(\sin(t^2) - \frac{2}{3}\sin^3(t^2) + \frac{\sin^5(t^2)}{5}\right)+C} = \frac{1}{2}\left(w - \frac{2}{3}w^3 + \frac{w^5}{5}\right) + C$$

#15) $\int \cot x \cos^2 x \, dx = \int \frac{\cos x}{\sin x} \cdot \cos^2 x \, dx$

$$= \int \frac{(1-\sin^2 x)}{\sin x} \cdot \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int \frac{1-u^2}{u} \, du$$

$$= \int \left(\frac{1}{u} - u\right) du$$

$$= \ln|u| - \frac{u^2}{2} + C$$

$$= \ln|\sin x| - \frac{\sin^2 x}{2} + C$$

#24) Evaluate $\int (\tan^2 x + \tan^4 x) \, dx$.

$$\int (\tan^2 x + \tan^4 x) \, dx = \int (1 + \tan^2 x) \tan^2 x \, dx$$

$$= \int \tan^2 x \cdot \sec^2 x \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$$= \int u^2 \, du$$

$$= \frac{u^3}{3} + C$$

$$= \frac{1}{3} \tan^3 x + C$$